Contents

Constructing of relative Prym varieties associated to a linear system on an Enriques surface

E. Arbarello

In a joint work with Giulia Saccà and Andrea Ferretti, we construct relative Prym varieties associated to a linear system on an Enriques surface. These are singular symplectic varieties whose smooth locus is a completely integrable Hamiltonian system. We describe these varieties in the hyperelliptic and non-hyperelliptic case. We show that their non-singular model is simply connected and possesses a unique holomorphic 2-form, up to scalars. The analysis of singularities in the hyperelliptic case leads to Nakajima's quiver varieties.

$\mathcal{W}$-constraints for the total descendant potential of a simple singularity

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Simple singularities are classified by Dynkin diagrams of type ADE. Let $\mathfrak{g}$ be the corresponding finite-dimensional Lie algebra, and W its Weyl group. The set of $\mathfrak{g}$-invariants in the basic representation of the affine Kac–Moody algebra $\hat{\mathfrak{g}}$ is known as a $\mathcal{W}$-algebra and is a subalgebra of the Heisenberg vertex algebra $\mathcal{F}$. Using period integrals, we construct an analytic continuation of the twisted representation of $\mathcal{F}$. Our construction yields a global object, which may be called a W-twisted representation of $\mathcal{F}$. Our main result is that the total descendant potential of the singularity, introduced by Givental, is a highest weight vector for the $\mathcal{W}$-algebra. (Joint work with T. Milanov.)

Derived Poisson Structures

Yu. Berest
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We introduce the notion of a derived Poisson structure on an associative algebra A. This structure is characterized by the property of being the weakest structure on A that induces natural (graded) Poisson structures on the derived moduli spaces of finite-dimensional representations of A. A derived Poisson structure on A gives rise to a graded (super) Lie algebra structure on the full cyclic homology HC(A) and can be viewed as a higher homological extension of noncommutative Poisson structures in the sense of M. Kontsevich.

Algebraic ansatz for heat equation and integrable polynomial dynamical systems

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We will discuss the ansatz that reduces the heat equation to a homogeneous polynomial dynamical system. For any such system in the generic case we obtain a nonlinear ordinary differential equation and algorithm for constructing a solution of this system. As result we have the corresponding solution of the heat equation. We give the full classification of nonlinear ordinary differential equations that arise from our ansatz.

The talk is based on recent joint works with E. Yu. Bunkova. Main definitions will be given during the talk.

Integrals of ψ-classes over double ramification cycles

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Double ramification cycles are certain codimension g cycles in the moduli space $\overline{\mathcal{M}}_{g,n}$ of stable genus g curves with n marked points. They have proved to be very useful in the study of the intersection theory of $\overline{\mathcal{M}}_{g,n}$. In my talk I will explain that integrals of arbitrary monomials in ψ-classes over double ramification cycles have an elegant expression in terms of vacuum expectations of certain operators that act in the infinite wedge space.

The talk is based on a joint work with S. Shadrin, L. Spitz and D. Zvonkine.

The Hirota equation for the descendent potential of orbifold CP^1 and the Lax formulation of bigraded Toda hierarchy

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The total descendent potential D associated, by the Givental formula, with the calibrated Frobenius manifold of Laurent polynomials is conjectured to coincide with the Gromov-Witten potential of a $\mathbb{C}P^1$ orbifold. Milanov and Tseng proved that the potential D is a solution of an Hirota quadratic equation, defined in terms of vertex operators whose coefficients are obtained from singularity theory, and conjectured that such Hirota equation is equivalent to the Lax formulation of the bigraded Toda hierarchy, introduced previously. After briefly reviewing the statement of Milanov-Tseng and the Givental formula, and we show how to obtain the bigraded Toda Lax equations, therefore proving the Milanov-Tseng conjecture, and we point out analogies and differences between the bigraded Toda hierarchies and the well-known Gelfand-Dickey reductions of KP. Based on joint work with J. van de Leur.

Local and non-local Poisson vertex algebras and applications to the theory of integrable systems

A. De Sole

Lecture 1. We lay down the foundations of the theory of Poisson vertex algebras aimed at its applications to integrability of Hamiltonian partial differential equations.

Lecture 2. It is well known that the validity of the so called Lenard-Magri scheme of integrability of a bi-Hamiltonian PDE can be established if one has some precise information on the corresponding 1st variational Poisson cohomology for one of the two Hamiltonian operators. We will see how to compute the variational Poisson cohomology for any quasiconstant coefficient Hamiltonian operator with invertible leading coefficient.

Lecture 3. We develop a rigorous theory of non-local Hamiltonian structures, built on the notion of a non-local Poisson vertex algebra. As an application, we find conditions that guarantee applicability of the Lenard-Magri scheme of integrability to a pair of compatible non-local Hamiltonian structures.

An example the Ricci-flow on 4-dimensional Walker space

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The Ricci flow on the manifold endowed with family of the Riemann metrics $ds^2 = g_{ij}(\vec{x},t)dx^i dx^j$ which depend on the parameter t is governed by the equation

$$\partial_t g_{ij} = -2R_{ij}, \tag{1}$$

where $R_{ij} = R_{ij}(\vec{x},t)$ -is the tensor of Ricci of the metric .

We consider the equation (1) to the $4D$- manifold with coordinates $\vec{x} = (x, y, z, \tau)$ with the family of Walker metric of the form

$$ds^2 = 2\,\frac{\partial^2}{\partial\tau^2}\rho(\vec{x},t)\,dx^2 - 4\,\frac{\partial^2}{\partial\tau\partial z}\rho(\vec{x},t)\,dx\;dy + 2\,\frac{\partial^2}{\partial z^2}\rho(\vec{x},t)\,dy^2 + \\ +2\;dy\;d\tau + 2\;dx\;dz. \tag{2}$$

In this case from the system of equations (1) we derive the equation relative the function $\rho(\vec{x},t)$

$$\frac{\partial}{\partial t}\rho(\vec{x},t) - 2\,\frac{\partial^2}{\partial\tau\partial y}\rho(\vec{x},t) - 2\,\frac{\partial^2}{\partial x\partial z}\rho(\vec{x},t) + \\ +2\,\frac{\partial^2}{\partial\tau^2}\rho(\vec{x},t)\frac{\partial^2}{\partial z^2}\rho(\vec{x},t) - 2\,\left(\frac{\partial^2}{\partial\tau\partial z}\rho(\vec{x},t)\right)^2 = 0. \tag{3}$$

Remark that the equation (3) is generalization of corresponding equation which was considered in [1].

Properties of the Ricci-flow of the metrics (2) depend from solutions of the equation (3). Let us consider some examples. After the substitution $\rho(x,y,z,\tau,t) = H(x+y+z+\tau,t)$ the equation (3) takes the form of heat equation $\phi_t - 4\,\phi_{\xi\xi} = 0$, where $\xi = x+y+z+\tau$. In addition to the well-known decisions it has solutions that depend on solutions of the Chazy equation $\dddot{h}(t) - 24\ddot{h}(t) + 36(\dot{h}(t))^2 = 0$ [2] (and its generalizations) which can be useful in theory of the Ricci-flow of the metrics (2). As example of a more complex solution of equation (3) we can cite the solution of the form

$$\rho(x,y,z,\tau,t) = t^k H(\frac{y-A(x)}{t^2}, \frac{\tau}{t^m}, zt).$$

In this case the equation (3) is reduced to various types of p.d.e., which can be integrated by help of generalized method of Monge-Ampere transformations [1] or to the linear p.d.e. integrate by cascade method of Laplace.

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Discrete Hamiltonian Structure of Schlesinger Transformations

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Schlesinger transformations are algebraic transformations of a Fuchsian system that preserve its monodromy representation and act on the characteristic indices of the system by integral shifts. One of the main reasons for studying these transformations is the relationship between Schlesinger transformations and discrete Painleve' equations; this is also the main motivation behind our work. In this talk we show how to write an elementary Schlesinger transformation as a discrete Hamiltonian system w.r.t. the standard symplectic structure on the space of Fuchsian systems. We also show how such transformations reduce to discrete Painlevé equations by computing two explicit examples, d-$P\big(D_4^{(1)}\big)$ (or difference Painlevé V) and d-$P\big(A_2^{(1)*}\big)$. In considering these examples we also illustrate the role played by the geometric approach to Painlevé equations not only in determining the type of the equation, but also in studying the relationship between different explicit forms of equations of the same type.

This is a joint work with Tomoyuki Takenawa (Tokyo University of Marine Science and Technology) and Hidetaka Sakai (The University of Tokyo).

Elliptic Calogero-Moser systems for complex crystallographic reflection groups

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To every irreducible finite crystallographic reflection group (i.e., an irreducible finite reflection group G acting faithfully on an abelian variety X), we attach a family of classical and quantum integrable systems on X (with meromorphic coefficients). These families are parametrized by G-invariant functions of pairs (T,s), where T is a hypertorus in X (of codimension 1), and s in G is a reflection acting trivially on T. If G is a real reflection group, these families reduce to the known generalizations of elliptic Calogero-Moser systems, but in the non-real case they appear to be new. We give two constructions of the integrals of these systems - an explicit construction as limits of classical Calogero-Moser Hamiltonians of elliptic Dunkl operators as the dynamical parameter goes to 0 (implementing an idea of Buchshtaber, Felder, and Veselov from 1994), and a geometric construction as global sections of sheaves of elliptic Cherednik algebras for the critical value of the twisting parameter. We also prove algebraic integrability of these systems for values of parameters satisfying certain integrality conditions. This is joint work with G. Felder, X. Ma, and A. Veselov.

Multivariable Lamé functions

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Multivariable Lamé functions are symmetric meromorphic solutions of the elliptic Calogero-Sutherland N-particle Schrödinger equation with integer parameter. They are closely related to affine Jack polynomials at the critical level. In the case of two particles they reduce to Lame' polynomials. I will review this classical theory and discuss extensions of the results to the multiparticle case.

The quad graph equation with a nonstandard generalized symmetry structure.

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The equation

$$u_{n+1,m+1}(u_{n,m} - u_{n,m+1}) - u_{n+1,m}(u_{n,m} + u_{n,m+1}) + 1 = 0 \qquad (1)$$

is found in article [1]. In those artcile is shown that eq.(1) have two generalized symmetry in different directions:

$$\frac{d}{dt_1}u_{n,m} = h_{n,m}h_{n-1,m}(a_n u_{n+2,m} - a_{n-1}u_{n-2,m}), \qquad (2)$$

$$\frac{d}{dt_2}u_{n,m} = (-1)^n \frac{u_{n,m+1}u_{n,m-1} + u_{n,m}^2}{u_{n,m+1} + u_{n,m-1}}, \qquad (3)$$

where $h_{n,m} = 1 - 2u_{n+1,m}u_{n,m}, \quad a_{n+2} = a_n$. One can see that n is an outer parameter in eq. (**??**), and this equation is really a known 1+1-dimensional autonomous equation of the Volterra type. In the case of eq. (2), we have the essentially non-autonomous Itoh-Narita-Bogoyavlensky equation with two-periodic coefficient a_n.

We show that eq. (3) can be rewritte as Gerdjikov-Ivanov-Tsuchida system [2] for odd and even $u_{n,m}$. We find Lax pairs for equations (1,2,3) in the form:

$$\Psi_{n+2,m} = N_{n,m}\Psi_{n,m}, \quad \Psi_{n,m+1} = M_{n,m}\Psi_{n,m}$$

$$\frac{d}{dt_1}\Psi_{n,m} = A_{n,m}\Psi_{n,m}, \quad \frac{d}{dt_2}\Psi_{n,m} = B_{n,m}\Psi_{n,m},$$

where $\Psi_{n,m}$ – vector function, $A_{n,m}, B_{n,m}, N_{n,m}, M_{n,m}$ – 2×2 matrtixes.

This work is a collaboration with A.V. Mikhailov and R.I. Yamolov.

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On a Class of Ferromagnetic Type Integrable Equations on A.III-type Symmetric Spaces

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For a class of integrable nonlinear differential equations related to **A.III**-type symmetric spaces the soliton solutions and the integrals of motion are studied. The class contains the following 2-component system (related to the symmetric space $SU(3)/S(U(1)\times U(2))$) of nonlinear evolution equations:

$$\begin{array}{l} \mathrm{i}u_t + u_{xx} + (uu_x^* + vv_x^*)u_x + (uu_x^* + vv_x^*)_x u = 0 \\ \mathrm{i}v_t + v_{xx} + (uu_x^* + vv_x^*)v_x + (uu_x^* + vv_x^*)_x v = 0 \end{array}, \qquad (1)$$

where the smooth complex-valued functions $u(x,t)$ and $v(x,t)$ satisfy the algebraic constraint $|u|^2+|v|^2=1$. It is associated with a Lax operator (for the time-evolution) *polynomial* in the spectral parameter λ and generalising the classical Heisenberg ferromagnetic equation.

The construction of the soliton solutions is based on the Zakharov-Shabat dressing method. Two classes of soliton solutions associated with the Lax operator are constructed.

The construction of the integrals of motion for the above class of equations is based on the direct approach, proposed by Drinfeld and Sokolov.

Based on a joint work with Vladimir S. Gerdjikov (Sofia), Alexander V. Mikhailov (Leeds) and Tihomir I. Valchev (Sofia).

Affine and Finite Lie Algebras and Integrable Toda Field Equations on Discrete Space-Time

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Systems of partial differential equations of the form

$$r^i_{x,y} = e^{\sum_{j=1}^{j=N} a_{ij} r^j}, \quad i = 1, 2, ...N, \qquad (1)$$

called generalized two-dimensional Toda lattices have very important applications in Liouville and conformal field theories. Here the matrix $A = \{a_{i,j}\}$ is the Cartan matrix of an arbitrary finite or affine Lie algebra. It is known that in the former case system (1) is Darboux integrable while in the latter case – S-integrable. The widely known Drinfel'd-Sokolov formalism allows one to construct the Lax representation for the system (1) in terms of the Lie algebra canonically associated with the corresponding Cartan matrix A.

The problem of finding discrete versions of the system is intensively studied. Recently, in [1] integrable differential-difference analog of system (1) was suggested

$$v^i_{1,x} - v^i_x = e^{\sum_{j=1}^{j=i-1} a_{i,j} v^j + \sum_{j=i+1}^{j=N} a_{i,j} v^j_1 + \frac{1}{2} a_{i,i}(v^i + v^i_1)}, \quad i = 1, 2, ..., N. \tag{2}$$

Here the functions $v^j = v^j(n, x)$, $j = 1, ..., N$ are the searched field variables. The subindex denotes a shift of the discrete variable n or the derivative with respect to x: $v^j_k = v^j(n + k, x)$ and $v^j_x = \frac{\partial}{\partial x} v^j(n, x)$.

In the present talk we discuss on the problem of further discretization of system (1), i.e. the problem of finding a rule allowing to assign to any Cartan matrix a system of integrable difference-difference equations approximating in the continuum limit system (2) and therefore system (1). The problem of discretization is important from physical viewpoint, they might have applications in discrete field theory and in quantum physics. They can also be regarded as difference schemes in numerical computations.

In [2] a fully discrete integrable version of system (1) is found:

$$e^{-u^i_{1,1} + u^i_{1,0} + u^i_{0,1} - u^i_{0,0}} - 1 =$$
$$= e^{\sum_{j=1}^{j=i-1} a_{i,j} u^j_{0,1} + \sum_{j=i+1}^{j=N} a_{i,j} u^j_{1,0} + \frac{1}{2} a_{i,i}(u^i_{0,1} + u^i_{1,0})}, \quad i = 1, 2, ..., N, \tag{3}$$

which evidently approximates (1) and (2). Here $u^j = u^j(n, m)$, $j = 1, 2, ...N$, is a set of the field variables. The subindex indicates shifts of the arguments n, m as follows $u^j_{i,k} := u^j(n + i, m + k)$.

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Short Lenard Chains and Integrable Systems

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It is well- known that certain classes of integrable PDE's are organized in hierarchies, called Lenard chains, defined by torsionless recursion operators. This property is lost in the dispersionless limit. In this case the notion of " short Lenard chain " seems to be more appropriate. In the talk I will discuss the interplay of this notion with the theory of Frobenius manifolds, and I will present a concrete example, related to a former work by Gepner.

On integrable classes of surfaces in Euclidean space

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We shall discuss methods and results of searching for integrable classes of surfaces in Euclidean space. The criterion is that the Gauss–Mainardi–Codazzi equations possess a zero curvature representation depending on a nonremovable parameter. We depart from the always-existing zero curvature representation equivalent to the Gauss–Weingarten system, and ask whether it can be extended to an infinite power series in the spectral parameter. The answwer to this question is provided in terms of a computable cohomological obstruction. Among the results obtained so far are known, unknown, and forgotten classes, which will be presented along with the related open problems.

Gaudin model and Cactus group

L. Rybnikov

Cactus group is the fundamental group of the real locus of the Deligne-Mumford moduli space of stable rational curves. We define an action of this group on the set of Bethe vectors of the Gaudin magnet chain (for Lie algebra sl_2) and relate this to the Berenstein-Kirillov group of piecewise linear transformations of the Gelfand-Tsetlin polytope. Some conjectures generalizing this construction will be discussed.

Local version of the spectral curve topological recursion

S. Shadrin

We explain a version of the topological recursion procedure of Eynard and Orantin for a collection of isolated local germs of the spectral curve. Under some conditions we can identify the n-point functions computed from spectral curve with the Givental formula for the ancestor formal Gromov-Witten potential. In particular, this way we prove a conjecture of Norbury and Scott on a particular spectral curve reproducing the stationary sector of the Gromov-Witten theory of the projective line.

The talk is based on a joint work with P. Dunin-Barkowski, N. Orantin, and L. Spitz.

Integrability of generalized pentagram maps and cluster algebra

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Talk 1. Introduction to cluster algebras and planar networks.

Cluster algebras were introduced by S.Fomin and A.Zelevinsky in 2001 in order to describe total positivity and dual Lusztig canonical basis. Rings of regular function on double Bruhat cells of simple Lie groups, Grassmannians, Teichmueller spaces, etc are equipped with the cluster algebra structure. Cluster algebras have compatible Poisson and, dually, presymplectic structures. In the talk I will discuss the definition of cluster algebra with examples. We will also discuss planar networks in the disk and in the annulus and cluster algebras associated with them.

Talk 2. Cluster algebras and complete integrability.

In this talk I will describe two connections between completely integrable systems and cluster algebras. In the first example cluster transformations play the role of Baecklund-Darboux transformations between different realizations of classical open Coxeter-Toda lattices. As the second example we consider pentagram maps. The pentagram map that associates to a projective polygon a new one formed by intersections of short diagonals was introduced by R. Schwartz and was shown to be integrable by V. Ovsienko, R. Schwartz and S. Tabachnikov. We prove complete integrability of pentagram map using theory of cluster transformations.

Both talks are based on joint works with M.Gekhtman, A.Vainshtein, and S.Tabachnikov

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Rational r-matrices, higher rank Lie algebras and integrable proton-neutron BCS models

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We consider integrable cases of pairing BCS hamiltonians containing several types of fermions. We show that there exist three classes of such the integrable models associated with classical rational r-matrices and Lie algebras $gl(2m)$, $sp(2m)$ and $so(2m)$ correspondingly. We diagonalize the constructed hamiltonians by means of the Bethe ansatz. In the partial case of two types of fermions ($m = 2$) the obtained models may be interpreted as $N = Z$ proton-neutron integrable models. In particular, in the case of $sp(4)$ we recover the famous integrable proton-neutron model of Richardson.

Mini-course

V. Sokolov

Lecture 1 "Symmetry approach to classification of integrable PDEs"

Basic notions of the standard symmetry approach to integrability and first classification results are described. New classification problems related to integrable isotropic and anisotropic vector evolution equations are considered.

Lecture 2 "Non-associative algebras and polynomial integrable systems"

We establish relations between integrable multi-component generalizations of known integrable models such as the Burgers, KdV, MkdV and NLS equations and algebraic structures of Jordan algebra type.

Lecture 3 "Integrable ODEs with matrix variables"

We present matrix generalizations of classical integrable PDEs and ODEs. We consider a special class of Poisson brackets related to ODE systems with matrix variables, investigate general properties of quadratic brackets of this type and associated double Poisson brackets.

Integral operator solution of the Yang-Baxter equation based on the elliptic beta integral

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A general solution of the Yang-Baxter equation is constructed as an integral operator with an elliptic hypergeometric kernel acting in the space of functions of two complex variables. It intertwines the product of two standard L-operators associated with the Sklyanin algebra (an elliptic deformation of sl(2)). This R-matrix is constructed from three operators generating the permutation group of four parameters entering L-operators. Validity of the Coxeter relations (including the star-triangle relation) is based on the elliptic beta integral evaluation and corresponding Bailey lemma. This is a joint work with S. Derkachov.

The principle realization of $D_n^{(1)}$ and W-constraints

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Bakalov and Milanov showed in a recent publication that the total descendant potential of an A, D or E type singularity is a highest weight vector for the corresponding W-algebra. In this talk I will approach this topic for type D. I will obtain the principle realization of the basic module of type $D_n^{(1)}$ as a certain reduction of a representation of D_∞. The reduction of the corresponding DKP-type hierarchy gives Hirota bilinear

equations for the corresponding tau functions. This gives an equivalent but slightly different formulation of Kac-Wakimoto D_n principle hierarchy. This approach has 3 advantages. (1) there is a Lax type formulation for this hierarchy; (2) there is a Grassmannian formulation for this reduced hierarchy; (3) one can show that the string equation generates part of the W-algebra constraints. This makes it possible to describe -at least part of- the W-algebra action on the corresponding Grassmannian.

Limits of integrable Hamiltonians on semisimple Lie algebras

E.B. Vinberg

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Let g be a semisimple Lie algebra, and let $P(g)$ be the corresponding Poisson algebra. With each regular element $a \in g$, the argument shift method associates a commutative subalgebra $F(a) \subset P(g)$, whose transcendence degree is maximal possible, i.e., is equal to the dimension of a Borel subalgebra of g. When a tends to a singular element in a proper way, the subalgebra $F(a)$ tends to some commutative subalgebra of the same transcendence degree. The cases when a tends to a singular element remaining in the same Cartan subalgebra, were investigated in old works of the speaker (1990) and V.V. Shuvalov (2002). Some other cases will be discussed in the talk. An interesting problem is to describe the variety of integrable quadratic Hamiltonians arising in this way.

Lenard scheme for multi-Hamiltonian systems

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Lenard scheme is used to generate infinitely many symmetries and conservation laws for multi-Hamiltonian systems. In this talk, I'll formulate the easily verified conditions for compatible weakly nonlocal Hamiltonian and symplectic operators to generate local hierarchies, and sketch the proof in the context of variational complex. I'll give examples for both continuous and discrete integrable systems including two-dimensional periodic Volterra chain and the discrete Adler's equation.

Tau functions and Virasoro symmetries for Drinfeld-Sokolov hierarchies

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For every affine Kac-Moody algebra, Drinfeld and Sokolov constructed a hierarchy of integrable systems that generalize the Korteweg-de Vries equation. In this talk, we construct tau function of each Drinfeld-Sokolov integrable hierarchy in a uniform way and represent Virasoro symmetries for the hierarchy as linear/nonlinear actions on the tau function. We also prove that, whenever the affine Kac-Moody algebra is simply-laced or twisted, the tau functions of the Drinfeld–Sokolov hierarchy coincide with the solutions of the corresponding Kac–Wakimoto hierarchy from the principal vertex operator realization of the affine algebra.

Properties of the series solution for Painlevé I

F. Zullo

Kent University

We present some observations on the asymptotic behaviour of the coefficients in the Laurent series expansion of solutions of the first Painlevé equation. For the general solution, explicit recursive formulae for the Taylor expansion of the tau-function around a zero are given, which are natural extensions of analogous formulae for the elliptic sigma function, as given by Weierstrass. Numerical results on a solution with pentagonal symmetry are also presented.