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ABSTRACTS

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Contents

Universal G-spaces in the sense of R. Palais

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We present the survey of the theory of universal G-spaces in the sense of R. Palais

On non-immersibility of $\mathbb{R}P^{10}$ to $\mathbb{R}^{15}$

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B.J. Sanderson noted that for $k < n$ the projective space $\mathbb{R}P^k$ is immersible in $\mathbb{R}^n$ if and only if the tangent bundle $T\mathbb{R}P^n$ admits k linearly independent vector fields over $\mathbb{R}P^k$ [1, Lemma (9.7)]. Using this remark, P.F. Baum and W. Browder proved that $\mathbb{R}P^{10}$ can not be immersed to $\mathbb{R}^{15}$ [1, Corollary (9.9)] by showing that the tangent bundle $T\mathbb{R}P^{15}$ does not admit 9 linearly independent vector fields over $\mathbb{R}P^{10}$ [1, Thm. (9.5)]. We present a new proof of this last statement based on U. Koschorke' singularity approach [2].

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Classification of low complexity knots in the thickened torus

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We compose the table of knots in the thickened torus $T \times I$ which have diagrams with ≤ 4 crossing points. The knots are constructed by a three-step enumeration. First we enumerate regular graphs of degree 4, then for each graph we enumerate all corresponding knot projection, and after that we construct the corresponding minimal diagrams. Several known and new tricks made possible to keep the process within reasonable limits and offer a rigorous theoretical proof of the completeness of the table. For proving that all knots are different we use a generalized version of the Kauffman polynomial.

Cutting the same fraction of several measures

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The famous "ham sandwich" theorem of Stone, Tukey, and Steinhaus asserts that every d absolutely continuous probability measures in $\mathbb{R}^d$ can be simultaneously partitioned into equal parts by a single hyperplane.

M. Kano and S. Bereg raised the following question (in the planar case): If we are given $d+1$ measures in $\mathbb{R}^d$ and want to cut the same (but unknown) fraction of every measure by a hyperplane then what assumptions on the measures allow us to do so? Certainly, additional assumptions are required because if the measures are concentrated near vertices of a d-simplex then such a fraction cut is impossible. A sufficient assumption is described below:

Definition. Let μ_0, μ_1, $\dots$, μ_d be absolutely continuous probability measures on $\mathbb{R}^d$ and let $\varepsilon \in (0, 1/2)$. Call the set of measures *ε-not-permuted* if for any halfspace H the inequalities $\mu_i(H) < \varepsilon$ for all $i =$

$0, 1, \ldots, d$ imply

$$\mu_i(H) \geq \mu_j(H), \text{ for some } i < j.$$

Theorem. *Suppose μ_0, μ_1, …, μ_d are absolutely continuous probability ε-not-permuted measures in $\mathbb{R}^d$ for some $\varepsilon \in (0, 1/2)$. Then there exists a halfspace H such that*

$$\mu_0(H) = \mu_1(H) = \cdots = \mu_d(H) \in [\varepsilon, 1/2].$$

We also consider a problem of cutting the same *prescribed* fraction of every measure, this time allowing cutting with a convex subset of $\mathbb{R}^d$.

Theorem. *Suppose μ_0, μ_1, …, μ_d are absolutely continuous probability measures on $\mathbb{R}^d$ and $\alpha \in (0, 1)$. It is always possible to find a convex subset $C \subset \mathbb{R}^d$ such that*

$$\mu_0(C) = \mu_1(C) = \cdots = \mu_d(C) = \alpha,$$

if and only if $\alpha = 1/m$ for a positive integer m.

On simply-paracompact space

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We introduce the class of simply-paracompact spaces as a generalization of paracompact spaces. A space X is called simply-paracompact if every open cover of X has a locally finite simply-open refinement. We characterize simply-paracompact spaces and study their basic properties. The relationships between simply-paracompact spaces and other well-known spaces are investigated.

The Schlesinger system and isomonodromic deformations of bundles with connections on Riemann sufraces

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Take a Riemann sphere a consider a fuchsian system on it:

$$dy = \sum_i \frac{B_i}{z - a_i} dzy,$$

where A_i are some $k \times k$ matrices, a_i, $i = 1, ..., n$ are point on the Riemann sphere and $y = (y_1, ..., y_k)^t$ - the column vector of unknowns.

We are changing a_i's in such a way that the monodromy is preserved. Then the sufficient and typically necessary for this is the Schlesinger system.

This system possesses numerous good properties. It is hamiltonian (there are explicit formulas for the hamiltonians), it possesses the Painleve property. If we take as a space of parameters not the set $\{(a_1, ..., a_n),\ a_i \in \overline{\mathbb{C}},\ a_i \neq a_j\}$, but it's universal covering, we can prove the following: a set of parameters in which we cannot continue the deformation is a set of zeroes of some function τ, which is holomorphic on the hole space of parameters. Moreover, there exists an explicit formulae for $dln\tau$ (The Miwa formulae). From this formulae we can see, that τ turns out to be the generating functions for the hamiltonians.

In the case of a Riemann surface of higher genus it is natural to work with bundles with connections instead of equations. It is natural to take as the parameters of deformations the positions of singularities and the complex structure with a fixed bases in the fundamental group (i.e. a point in the Teichmuller space).

The main result of the talk is a natural description of isomonodromic deformations of bundles.

It turns out that one can describe such deformations by the same Schlesinger system as in the case of genus 0 plus some system of linear equations.

A counterexample to the Lando conjecture on intersection of spheres in 3-space

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Let $\mathcal{M}$ and $\mathcal{N}$ be two sets of the same number of disjoint circles in a sphere. Do there exist two polyhedral two-dimensional spheres in $\mathbb{R}^3$ such that their intersection is $\mathcal{M}$ in one of them and $\mathcal{N}$ in the other? *S. Lando conjectured* that the answer is "yes" for each $\mathcal{M}$ and $\mathcal{N}$. We shall give a counterexample to this conjecture. We shall also prove the following necessary and sufficient condition on $\mathcal{M}$ and $\mathcal{N}$ for existing of such intersecting spheres.

Suppose there are two unions of disjoint circles in a sphere. We shall say that they are *comparable* (in this sphere) if any path connecting two points of one union intersects the other union in an even number of points.

Suppose $\mathcal{M}$ is a union of disjoint circles in a sphere S. Clearly, closures of the connected components of $S-\mathcal{M}$ can be colored in two colors so that any two same colored components are not adjacent.

Theorem 1 *Let $\mathcal{M}$ and $\mathcal{N}$ be two unions of disjoint circles in a sphere S. Let $h : \mathcal{N} \to \mathcal{M}$ be a homeomorphism. Then there exist homeomorphisms $f, g : S \to \mathbb{R}^3$ such that $g|_{\mathcal{N}} = f \circ h$ and $f(S) \cap g(S) = f(\mathcal{M}) = g(\mathcal{N})$ iff $h(\partial A)$ and $h(\partial B)$ are comparable for any same colored closures of connected components A and B of $S - \mathcal{N}$.*

Algebraic properties of spherical nerve-complexes

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Consider a convex polytope P. Its facets cover its boundary and the nerve of this cover is denoted K_P. For a simple polytope P the complex K_P coincides with the boundary ∂P^* of a polar dual polytope. In this case K_P is a simplicial sphere and its Stanley–Reisner ring $\mathbf{k}[K_P]$ is known to be Cohen–Macaulay. The global problem is to describe the properties of a simplicial complex K_P and its Stanley–Reisner ring for general convex polytope P. Topological properties of K_P are put together in the definition of a spherical nerve-complex. In the talk a connection between the topology of a simplicial complex K and the depth of the ring $\mathbf{k}[K]$ will be shown. This connection yields the central result of the work:

$$\operatorname{depth} \mathbf{k}[K_P] = \dim P$$

for each convex polytope P.

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On points of compactifications of discrete spaces

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We consider the compactification BN of a countable discrete space N, constructed as Stone space of one Boolean algebra of N.

We prove the existing of three classes of points in remainder of this space: u-, ℓ- and $\ell_{\pi|M}$-points, prove that this classes are disjunct and that in $BN \setminus N$ there are points which are not u-, ℓ- or $\ell_{\pi|M}$-points. We also get a characteristics of points from specified classes in terms of centered systems of sets and prove some properties of closures of countable sets of this points.

The weakly density of superextension

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In the paper, cardinal properties of superextensions are investigated.

Let X be a topological space and λX its superextension [1].

Definition 1 [2]. The MLS $\xi \in \lambda X$ is called thin (TMLS) if it contains at least one finite element. We denote by $\lambda^* X$ the set of all TMLS of the space X

Definition 2. The MLS $\xi \in \lambda X$ is called compact (CMLS) if it contains at least one compact element.We denote by λ_c the set of all CMLS of X.

Example. There exists a MLS containing compact element such that it doesn't contain any finite element. Let R be the real line with the natural topology. If we consider following closed sets: $F_1 = [0,1]$, $F_2 = \{\frac{1}{2}\} \cup [2,+\infty)$, $\ldots$, $F_k = \{\frac{1}{k}\} \cup [k,+\infty)$, $\ldots$, it is clear that the system $\mu = \{F_1, F_2, ..., F_k, ...\}$ is linked. We'll fill it up to MLS ξ. It is easy to check that the system ξ doesn't contain any finite element.

Theorem 1. We have

1)$\pi w(\lambda_c X) = \pi w(X)$;

2)$\pi\chi(X) \leq \pi\chi(\lambda_c X)$;

3) If an infinite cardinal number τ is a caliber of a space X then τ is a caliber for $\lambda_c X$;

4) If X is an infinite Tychonoff space then

$wd(X) = wd(\lambda^* X) = wd(\lambda_c X) = wd(\lambda X)$

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Combinatorial types of polyhedra and Rakov conjecture

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One of the most important problems in studying combinatorial types of polyhedra is the enumeration of all convex polyhedra with a particular configuration of faces. By Steinitz's theorem it is equivalent to the enumeration of planar 3-connected graphs. Checking a graph for planarity and 3-connectivity is a relatively simple task, but checking for isomorphism is notoriously hard, which motivates to develop an algorithm to efficiently check planar 3-connected graphs for isomorphism.

In 2005, Rakov stated a conjecture that two 3-connected planar graphs are isomorphic iff. they have the same number of vertices of each degree and also the same number of spanning trees. Using Maple, he checked his conjecture for $N \leq 8$.

This work's aim is programming directly the algorithms above (using Java) and checking the conjecture for greater N. More specifically, we calculate the number of 3-connected planar graphs with different Rakov invariants, and compare that to the number of all non-isomorphic 3-connected planar graphs (which was computed in Engel's work for $N \leq 12$). The first number is less or equal to the second, and if it's less - the conjecture is refuted.

The result of this work is the refutation of Rakov conjecture. Besides, the counterexample was found with only 8 vertices, implying that there was probably a mistake in Rakov's work. First, the number of 3-connected planar graphs with different Rakov invariants was computed. Even at $N = 8$, it was less than the number of non-isomorphic 3-connected planar graphs. But a specific counterexample is always preferable, and it was found applying divisibility considerations to one of the Rakov invariants sets, and checking a suspicious set of graphs for isomorphism. This indeed yielded a counterexample.

Random polytopes with (0-1)-vertices

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Let M be the convex polytope, $X = extM$ - the set of its vertices. Two vertices x and y of X are called adjacent if the segment $[x, y]$ is an edge of the polytope M.

Polytope M is called a 2-neighborly, if its graph is complete, i.e. any two vertices are adjacent.

We denote by $P_{k,m}$, the probability that the convex hull of randomly selected k points from $\{0,1\}^m$ forms a 2-neighborly polytope. From the results of [1, 2] implies the following theorem.

Theorem.

1. Let $k = k(m) = \alpha_m(\frac{8}{5})^{\frac{m}{4}}$, where $\alpha_m \to 0$, then $P_{k,m} \to 1$.

2. Let $k = k(m) \geq (2+\epsilon)^{\frac{m}{2}}$, for some $\epsilon > 0$, then $P_{k,m} \to 0$.

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Nakajima's quiver varieties and combinatorial identities

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In the talk I will explain how different combinatorial identities can be proved using geometrical constructions in the theory of Nakajima's quiver varieties. This identities include a quantum generalization of the Macmahon's formula and infinite product expansions of certain generating functions for statistics on Young diagrams.

The (dis)connectedness of products in the box topology

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This is a joint talk with A. Karassev.

It is well known that the product R^ω of countably many copies of the real line R, endowed with the box topology, is disconnected. Recall [W, Theorem 1.3] that this result can be generalized as follows. Let $X_\alpha, \alpha \in A$, be an infinite system of nondegenerated completely regular T_1-spaces and $\prod^b_{\alpha \in A} X_\alpha$ denote the product endowed with the box topology. Then the space $\prod^b_{\alpha \in \mathcal{A}} X_\alpha$ is disconnected.

This study is motivated by the following question. Let $X_\alpha, \alpha \in A$, be an infinite system of nondegenerated connected spaces. Under what conditions on the system is the space $\prod^b_{\alpha \in A} X_\alpha$ (dis)connected?

In view of the result above the question is meaningful for connected spaces with axioms lower than $T_{3\frac{1}{2}}$. In this talk we suggest two independent sufficient conditions on topological connected spaces which imply disconnectedness, and one sufficient condition which implies connectedness, of products of spaces endowed with the box topology. As an application of that we show that the product K^ω of countably many copies of Khalimsky line K, endowed with the box topology, is also disconnected. Moreover, we describe the connected components of the product.

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A classification of embeddings of non-simply connected 4-manifolds in 7-space

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Let N be a closed connected orientable 4-manifold with torsion free integral homology. The main result is *a complete readily calculable classification of embeddings* $N \to R^7$, in the smooth and in the piecewise-linear (PL) categories. Such a classification was earlier known only for simply-connected N, in the PL case by Boéchat-Haefilger-Hudson 1970, in the smooth case by the authors 2008 (arxiv:math/0808.1795). In particular, for $N = S^1 \times S^3$ we define geometrically a 1–1 correspondence between the set of PL isotopy classes of PL embeddings $S^1 \times S^3 \to R^7$ and the quotient set of $Z \oplus Z_6$ up to equivalence $(l, b) \sim (l, b')$ for $b \equiv b' \mod 2GCD(3, l)$. This particular case allows us to disprove the conjecture on the completeness of the Multiple Haefliger-Wu invariant, as well as Melikhov informal conjecture on the existence of a geometrically defined group structure on the set of PL isotopy classes of PL embeddings in codimension 3. For $N = S^1 \times S^3$ we identify the smooth isotopy classes of smooth embeddings with an explicitly defined quotient of $Z_{12} \oplus Z \oplus Z$.

A link between Italian and Russian mathematical schools: Arzelà and Alexandroff convergences

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In 1883 Arzelà solved the fundamental question of Real Analysis: what precisely must be added to pointwise convergence of a sequence of continuous functions to preserve continuity? He formulated a set of conditions which are both necessary and sufficient (namely "*convergenza a tratti*") for the continuity of the pointwise limit of a sequence of real valued continuous functions defined on a closed interval of the real line. This problem has been dominant for the last century and an active area of research in Analysis and General Topology. In 1948 P.S. Alexandroff solved the problem for a sequence of real valued continuous functions from a topological space

X (not necessarily compact) to a metric space Y. Alexandroff's idea is one of the most important in the field.

We discuss relations between these two modes of convergence.

Local τ-density of exponential spaces

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In the work it is shown that functors exp_n, exp_ω, exp_c preserve local τ-density of any topological T_1-space.

Let X be a topological T_1-space. We denote by $expX$ the set of all nonempty closed subsets of the space X. The family B of all sets of the form $O\langle U_1, U_2, ..., U_n\rangle = \{F : F \in expX,\ F \subset \bigcup U_i,\ F \cap U_i \neq \emptyset,\ i = 1, 2, ..., n\}$, where $U_1, U_2, ..., U_n$ is a sequence of open sets of the space X, generates a topology on the set $expX$. This topology is called the Vietoris topology. The set $expX$ with the Vietoris topology is called the exponential space or the hyperspace of X [1].

Let X be a topological T_1-space. We denote by exp_nX the set of all nonempty subsets of the space X containing no more than n elements, i.e. $exp_nX = \{F \in expX : |F| \leq n\}$. We assume that $exp_\omega X = \cup\{exp_nX : n = 1, 2, ..., n, ...\}$, $exp_cX = \{F \subset expX : F - compact\}$.

Definition 1. T_1-space X is called local τ-dense if each point $x \in X$ has a neighborhood Ox with the density τ.

If $\tau = \aleph_0$ - countable then the space X is locally separable.

Theorem 1. If a T_1-space X is locally τ-dense then so are spaces exp_nX, $exp_\omega X$, exp_cX.

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On generalizations of Fedorchuk's Normal Functor Theorem in category $\mathcal{P}$

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A well-known Katětov theorem states, that the hereditary normality of the cube of a compact space implies the metrizability of this space.

In 1989, the theorem was generalized by V.V. Fedorchuk:

Theorem 1. *If a compact Hausdorff space $\mathcal{F}(X)$ is hereditary normal for some normal functor $\mathcal{F}$ of degree ≥ 3, then X is metrizable.*

As T.F. Zhuraev showed in [3], the condition of the hereditary normality of $\mathcal{F}(X)$ in theorem 1 can be replaced with the condition of the hereditary normality of $\mathcal{F}(X)\setminus X$. The Fedorchuk's theorem, as well as Zhuraev's, was also generalized by A.P. Kombarov: the hereditary normality of $\mathcal{F}(X) \setminus X$ was relaxed to the weaker requirement of the hereditary $\mathcal{K}$-normality of $\mathcal{F}(X) \setminus X$.

All the above-mentioned results are true for normal functors acting in category Comp. Therefore it seems natural to extend these results to some wider classes of covariant functors: for instance, by considering the category $\mathcal{P}$ of paracompact $p-$spaces, which are exactly the full perfect preimages of metrizable spaces, and their perfect mappings.

In this connection we generalized the notion of a normal functor to the category $\mathcal{P}$ and proved the following theorem, which generalizes theorem 1:

Theorem 2. *Suppose that X is a paracompact $p-$space, $\mathcal{F}$ is a normal functor of degree ≥ 3 acting in category $\mathcal{P}$, and the space $\mathcal{F}(X)$ is hereditarily normal. Then X is a metrizable space.*

Furthermore, we obtained the generalization of Zhuraev's result, which also generalizes theorem 2:

Theorem 3. *Suppose that X is a paracompact $p-$space, $\mathcal{F}$ is a normal functor of degree ≥ 3 acting in category $\mathcal{P}$ and the space $\mathcal{F}(X) \setminus X$ is hereditarily normal. Then X is metrizable.*

Some Generalizations of Transversal Theorems

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In my talk I shall concentrate on some recent "colored" transversal theorems.

Let P be a family of sets. By $\tau(P)$ denote the least positive integer k such that there exists a set T of cardinality k which has a nonempty intersection with every $V \in P$.

The following concepts is a generalization of H. Hadwiger – H. Debrunner concept. Let p and q are integers with $p \geq q \geq 2$. We say that a collection $P_1, P_2, \dots P_p$ of families of sets has (p,q)-property when the following condition holds: if for every i, $1 \leq i \leq p$, we choose an arbitrary set $V_i \in P_i$, then it is possible to find q sets among $V_1, \dots V_p$ with a nonempty intersection.

The following theorem is a generalization of Hadwiger – Debrunner theorem and, at the same time, a generalization of Bárány – Lovász theorem.

Theorem 1. If a collection $P_1, P_2, \dots P_p$ of families of compact convex sets in $\mathbb{R}^d$ has (p,q)-property with $d+1 \leq q \leq p \leq \frac{d}{d-1}(q-1)$, then $t(P_i) \leq p-q+1$ for an index i, $1 \leq i \leq p$.

Now we will give a result in direction of (p,q)-problem for special families of convex sets.

Let V, W are compact convex sets in $\mathbb{R}^d$. We write $V \leq W$ if there exists a compact convex set U such that $W = V + U$. A family P of compact convex sets in $\mathbb{R}^d$ is called monotone if for every V, $W \in P$ one of the relations $V \leq W$ or $W \leq V$ holds.

Theorem 2. If a collection $P_1, \dots P_{m+1}$ of monotone families of compact convex sets in $\mathbb{R}^d$ has $(m+1, 2)$-property, then there exists i ,$1 \leq i \leq m+1$, such that $\tau(P_i) \leq a(d)m$ where $a(d)$ depends only on d

Pseudo-integrable billiards: an introduction

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We introduce a class of nonconvex billiards with a boundary composed of arcs of confocal conics. We present their basic dynamical, topological and arithmetic properties. We study their periodic orbits and establish a local Poncelet porism. This research is done jointly with M. Radnović.

Integrable hierarchies of topological type from dressing transformations

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We consider the class of hierarchies of integrable PDEs satisfying topological recursion coming from Deligne-Mumford moduli spaces of stable algebraic curves. Many classical examples like Korteweg - de Vries, nonlinear Schroedinger, Toda lattice equations belong to this class but there are many new hierarchies depending on continuous parameters. We construct a big family of such hierarchies with the help of the well known dressing transformations and their quantization.

On matched diagrams of knots

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A matched diagram of a knot is a plane knot diagram whose crossings can be matched into simplest pairs having the form of a braid on two srings with one full twist. The problem raised by Jozef Przytycki in 1987 was to find a knot which does not have a matched diagram. In this talk, based

on a joint work with M.Shkolnikov (see arxiv:1105.1264), we will explain why, for example, the pretzel knot $P_{3,3,-3}$ cannot be drawn by a matched diagram. The proof relies on the construction of a special Seifert surface of the knot from its matched diagram, then writing out the Seifert matrix for a cleverly chosen basis o cycles and, finally, arriving at an Alexander matrix whose elements are polynomials in the combination $t + t^{-1}$. It follows that the second Alexander ideal of such a knot is an ideal of the ring $\mathbb{Z}[t, t^{-1}]$ that can be generated by a set of polynomials in the variable $t + t^{-1}$. However, it is known that the second ideal of the knot $P_{3,3,-3}$ is $\langle 3, t + 1 \rangle$, and it is readily seen that this ideal does not allow for a set of generating polynomials in $t + t^{-1}$ only.

The signature and its generalizations for open manifolds

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For a closed oriented $4k$-manifold M there are 4 standard definitions of the signature,

1) the signature of the combinatorial intersection form,

2) the signature of the analytical intersection form,

3) the index of the signature operator,

4) the integrated L-polynomial.

All these 4 numbers coincide.

If M is open, of bounded geometry with a uniform triangulation, then these numbers are not defined in general and if they are defined, they do not coincide. We give simple examples for this.

We extend in special cases the definitions above to open manifolds, exhibit their relations, the geometrical meaning, applications to bordism theory and give – following Higson and Roe – much more general definitions of the signature with values in certain K-groups for C^*-algebras which are defined for geometric Hilbert Poincaré complexes.

Ring of flag vectors of convex polytopes

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The famous g-theorem gives necessary and sufficient condition for a vector $(f_0, \dots, f_n) \in \mathbb{Z}^{n+1}$ to be the face-vector of an n-dimensional simple convex polytope. The analogous problem for flag numbers of convex polytopes is one of the most striking problems in the polytope theory and is opened even in dimension 4. The ring of convex polytopes with the algebra of face operators on it invented by Victor Buchstaber gives a new effective approach to the problem of flag vectors. The free abelian group $\mathfrak{P}$ generated by combinatorial convex polytopes with the multiplication given by the direct product is called *a ring of convex polytopes*. We can define the flag number of integer combination of polytopes by linearity. There is a graded ring $\mathfrak{F} = \mathfrak{P}/\sim$, where $P \sim Q$ if and only if P and Q have equal flag numbers and the grading is induced by doubled dimension of polytopes. This ring is called *a ring of flag vectors*.

Theorem 1. The ring $\mathfrak{F}$ can be realized as a graded subring in the ring $\mathrm{Qsym}[\alpha]$, $\deg \alpha = 2$, of polynomials over the ring of quasi-symmetric functions.

Theorem 2. The ring $\mathfrak{F} \otimes \mathbb{Q}$ is a graded ring of polynomials with $\dim(\mathfrak{F} \otimes \mathbb{Q})^{2n}$ equal to the n-th Fibonacci number ($c_0 = c_1 = 1$, $c_{n+1} = c_n + c_{n-1}$). This gives the expansion of the generating function of Fibonacci numbers $\frac{1}{1-t-t^2} = \sum_{n=0}^{\infty} c_n t^n$ into the infinite product $\prod_{n=1}^{\infty} \frac{1}{(1-t^n)^{k_n}}$, where k_n is the number of multiplicative generators of $\mathfrak{F} \otimes \mathbb{Q}$ in dimension $2n$. We have: $k_1 = k_2 = 1$.

Using the fact that $\mathrm{Qsym} \otimes \mathbb{Q}$ is a graded ring of polynomials in the set of multiplicative generators indexed by the Lyndon words, we obtain:

Corollary. We have: k_n, $n > 2$, is equal to the number of Lyndon words of degree n consisting of odd numbers. We give the conditions on flag numbers of k_n elements in $\mathfrak{F}^{2n}$ to be the set of $2n$-dimensional multiplicative generators.

J.Fine suggested the construction that associates to a convex polytope P a ***cd**-index* – the non-commutative polynomial in two variables $\boldsymbol{c}$ and $\boldsymbol{d}$, which captures all the information on the flag vector of P. It was shown by R.Stanley that the coefficients of the ***cd***-index are nonnegative. Consider the operators $\mathcal{C} = 2\mathrm{pyr} - \mathrm{bipyr}$ and $\mathcal{D} = \mathrm{pyr} \circ \mathrm{bipyr} - \mathrm{bipyr} \circ \mathrm{pyr}$, where $\mathrm{pyr}(P)$ is the pyramid and $\mathrm{bipyr}(P)$ is the bipyramid over P.

Theorem 3. The images of the elements $\mathcal{W}(\mathrm{pt})$, where $\mathcal{W}$ is a word in $\mathcal{C}$ and $\mathcal{D}$ and pt is a point, under the natural projection $\pi\colon \mathfrak{P} \to \mathfrak{F}$ form a basis in $\mathfrak{F}$ and the representation of $\pi(P)$ in this basis coincides with the ***cd***-index.

The talk is based on the joint work: V.M.Buchstaber, N.Yu.Erokhovets, *Polytopes, Fibonacci numbers, Hopf algebras, and quasi-symmetric functions*, Russian Math. Surveys, 66:2 (2011), 271–367.

Homotopy bundle gerbes and higher twisted K-theory

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It is well-known that twistings in complex K-theory over a compact space X are classified by homotopy classes of maps $X \to B(\mathbb{Z}/2\mathbb{Z}\times BU_{\otimes}) \simeq K(\mathbb{Z}/2\mathbb{Z}, 1) \times K(\mathbb{Z}, 3) \times BBSU_{\otimes}$. The twisted K-theory corresponding to "abelian" twistings from $H^1(X, \mathbb{Z}/2\mathbb{Z}) \times H^3(X, \mathbb{Z})$ has been intensively studied during the last decade while higher twistings from $[X, BBSU_{\otimes}]$ have not attracted much attention partly because there is no known appropriate geometric model for them. In my talk I shall discuss an approach for higher twistings of finite order based on the monoid of endomorphisms of the direct limit of complex matrix algebras. I shall also discuss a homotopy coherent version of bundle gerbes which combines the idea of a bundle gerbe with Wirth-Stasheff's local description of fibre bundles with structure monoid by homotopy transition cocycles. (Joint work with Thomas Schick.)

Cascade search of singularities of mappings between metric spaces

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The subject matter of the talk concerns the search of such singularities of (collections of) mappings between metric spaces as: zeros of nonnegative real functionals, common fixed points, coincidence point, common roots etc.

The cascade search methods were proposed and studied in [1] and other author's papers. A multicascade on a metric space X is a multivalued discrete dynamic system on X, that is an action on X of the semigroup $(\mathbb{Z}_{\geq 0}, +)$. The action of $1 \in \mathbb{Z}_{\geq 0}$ is a set-valued mapping called a generator of the multicascade. A non-negative set-valued real functional φ is called (α, β)-*search* on X, $0 < \beta < \alpha$, if $\forall x \in X$, $\exists x' \in X$ such that $\rho(x, x') \leq \frac{\varphi_*(x)}{\alpha}, \varphi_*(x') \leq \frac{\beta}{\alpha} \cdot \varphi_*(x)$. Here $\varphi_*(x)$ stands for $\inf\{y | y \in \varphi(x)\}$. With the help of a given search functional φ, the cascade search principle enables one to construct so called *search multicascade* on X which limit set is $Nil(\varphi) = \{x \in X | 0 \in \varphi(x)\}$. Essential generalizations of several known results were obtained as consequences of that principle and stability problems were also considered.

In this talk we present new versions of the cascade search principle using the more wide class of functionals than the search ones. The other new development concerns local versions of cascade search methods which allow one to realize a cascade search locally, within a given neighbourhood of the starting point. Some applications of the mentioned new results to the cascade search of singularities of finite collections of set-valued mappings between metric spaces will be presented as well.

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Exact values of complexity for some infinite series of hyperbolic manifolds with boundary

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Exact values of complexity are known only for few infinite series of 3-manifold. We present results on complexity for two infinite series of hyperbolic 3-manifolds with boundary. The first is a family of Paoluzzi – Zimmermann manifolds from [1], and the second is a family of their generalizations.

For every $n \geqslant 3$ consider an n-gonal bipyramid which is the union of pyramids $NL_0L_1 \dots L_{n-1}$ and $SL_0L_1 \dots L_{n-1}$ along the common n-gonal base $L_0L_1 \dots L_{n-1}$. Let k be such integer that $0 \leqslant k < n$. The first family corresponds to the case $\gcd(n, 2-k) = 1$, and the second – to the case $\gcd(n, 2-k) = 2$. We identify the faces of $\mathcal{B}_n$ in pairs: for each $i = 0, \dots, n-1$ the face $L_iL_{i+1}N$ gets identified with the face $SL_{i+k}L_{i+k+1}$ by a homeomorphism of faces (indices are taken $\mod n$ and the vertices are glued together in the order in which they are written). Identifications define the equivalence relations on the sets of faces, edges, and vertices of the bipyramid. It is easy to see that all the faces are split into pairs equivalent faces, all edges and all vertices become identified to a single edge resp. vertex (this is guaranteed by the above conditions on k). Denote the resulting identification spaces by $M^*_{n,k}$. It is an orientable pseudomanifold with one singular point. Cutting of a cone neighborhood of the singular point from $M^*_{n,k}$ we get a compact manifold $M_{n,k}$ with one boundary component.

Denote by $c(M_{n,k})$ the Matveev's complexity of $M_{n,k}$ which is defined as the minimum possible number of true vertices of an almost simple spine of $M_{n,k}$.

Theorem 1 [2, 3] *Suppose that* $\gcd(n, 2-k) = 1$ *or* $\gcd(n, 2-k) = 2$. *Then for every integer* $n \geqslant 4$ *we have* $c(M_{n,k}) = n$.

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Belt diameter of some class of space filling zonotopes

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In this talk we will discuss one combinatorial property of convex polytopes with centrally symmetric facets. For this class of polytopes we can define a notion of belt. For given d-dimensional polytope P and its face F of codimension 2 a *belt* $\mathcal{B}_P(F)$ is the set of all facets of P that are parallel to F. *Belt diameter* of P is the diameter of graph with vertices correspondent to pairs of opposite facets of P and edges connecting two vertices if and only if corresponding pairs of facets are in one belt. In other words, if we consider a surface of P as a city with facets as subway stations, and all belts as subway routes then belt diameter is the maximal number of route changes that we need to do in order to travel from any station to any other.

During this talk we will give a way how to obtain upper bounds for belt diameters of space-filling zonotopes. Also we give exact bounds for special case of zonotopes obtained from permutahedron by contraction of zone vectors. Namely we will point ideas of proofs of the following theorems.

Theorem. Belt diameter of d-dimensional space filling zonotope is not greater than $\lfloor \log_2 \frac{4}{5}d \rfloor$.

Theorem. If Z is a d-dimensional zonotope obtained from permutahedron by contraction of zone vectors then belt diameter of Z is not greater than 3 for $d \geq 8$ and not greater than 2 for smaller dimension. These bounds are sharp.

Why upper bounds for belt diameters are interesting and why we restrict to the case of space filling polytopes? One of the conjectures in the parallelohedra theory that deals with space filling polytopes is the Voronoi conjecture. The Voronoi conjecture claims that every *parallelohedron*, i.e. convex poltyope that tile Euclidean space with translations, is affine image of Dirichlet-Voronoi polytope for some lattice. One of the way to prove it for a single parallelohedron or for a family of parallelohedra is to construct an explicit affine transformation using belts of parallelohedra.

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Local liftability of tilings

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An old enough problem is to determine whether a given convex tiling C^d of $\mathbb{R}^d$ is a projection of some convex polyhedron P in $\mathbb{R}^{d+1}$ (C. Davis, F. Aurenhammer, P. McMullen). There is a number of useful and handy criteria of being such projection. Most of the criteria deal with global structure while usually we can construct just local objects (expecting them to join together later). We provide a new local criterion which could be of use in cases when we know just local structure well. This criterion connects (a bit unexpectedly) the topic with a topic of polytopal fans.

Tietze-Gleason theorem for binary G-spaces

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Let X be a topological space and let G be an arbitrary topological group.

Definition 1. A *binary action* of G on X is a continuous map $\alpha : G \times X^2 \to X$ such that

$$\alpha(gh, x_1, x_2) = \alpha(g, x_1, \alpha(h, x_1, x_2)),$$

$$\alpha(e, x_1, x_2) = x_2.$$

for all $g, h \in G$ and $x_1, x_2 \in X$.

By a *binary G-space* we mean a topological space X together with a binary action of G on X.

There is a natural binary action of the general linear group $GL(n, \mathbf{R})$ on a vector space $\mathbf{R}^n$ defined by

$$\alpha(A, \mathbf{x}, \mathbf{y}) = \mathbf{x} + A(\mathbf{y} - \mathbf{x}),$$

$A \in GL(n, \mathbf{R})$, $\mathbf{x}, \mathbf{y} \in \mathbf{R}^n$.

For any $g \in G$ let $\alpha_g : X^2 \to X$ a continuous map defined by

$$\alpha_g(x_1, x_2) = \alpha(g, x_1, x_2),$$

$x_1, x_2 \in X$.

Proposition. The map $g \to \alpha_g$ is homomorphism of a group G to the group of all invertible continuous binary operations of a topological space X.

Definition 2. A continuous map $f : X \to Y$ between binary G-spaces (G, X, α) and (G, Y, β) is called *equivariant map*, provided

$$f(\alpha(g, x_1, x_2)) = \beta(g, f(x_1), f(x_2))$$

for all $g \in G$ and $x_1, x_2 \in X$.

Theorem[Tietze-Gleason] Let G be a compact group and let $\rho : G \to GL(n, \mathbf{R})$ be a representation of a group G. If X is a normal binary G-space and A is a closed invariant subspace of X then any equivariant continuous map $\varphi : A \to \mathbf{R}^n$ has an equivariant extension $f : X \to \mathbf{R}^n$.

About a Stone space of one Boolean algebra

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We consider the Boolean algebra of the same type as algebra constructed by Bell, and the Stone space of this Boolean algebra. This space is a compactification of a countable discrete space N.

We prove that there are isolated points in remainder of this compactification, which are limits of some convergent sequences. We prove that a clopen subset of our space, which is homeomorphic to $\beta\omega$, is a closure of an union of finitely many antichains from N.

We construct two examples: a clopen subset of the remainder without isolated points, which is not homeomorphic to $\beta\omega \setminus \omega$; a subset of the remainder which is homeomorphic to $\beta\omega \setminus \omega$, but is not a clopen.

v

Coincidence of maps between two arbitrary spheres

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Given a map $f : M \to N$ where M, N are closed manifolds, one can ask if the map f can be deformed to a map g such that the pair (f, g) is coincidence free, i.e. $\{x \in M \mid f(x) = g(x)\}$ is empty. In the case where M and N have the same dimension, this problem is well understood. In the study of this question when the dimension of M is greater than the dimension of N, a new question arises as follows: Assuming that a map g as above exists, can we find g' which is a small deformation of f and has no coincidence with f? It turns out that the answer of this problem is always "yes" if $dim(M) < 2dim(N) - 2$, but there are examples where the answer is "no" in case $dim(M) \geq 2dim(N) - 2$. The purpose of the talk is to describe the state of the art of this question when the manifolds M and N are spheres. We describe families of examples where the answer of the problem is "no" and the relation of this problem with the strong Kervaire invariant one problem. Also some known elements of the homotopy group of the sphere are analyzed, but certainly not all. Few basic references closely related with the talk follow below.

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L-spaces and left-orderability

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We will discuss evidence for the conjecture that a rational homology 3-sphere is an L-space if and only if its fundamental group is not left-orderable. This is joint work with Steve Boyer and Liam Watson.

Homotopy Rigidity of the Functor $\Sigma\Omega$

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The main problem of this talk is the study of the homotopy rigidity of the functor $\Sigma\Omega$. Our solution to this problem depends heavily on new decompositions of looped co-H-spaces. I shall start by recalling some classical homotopy theoretical decomposition type results. Thereafter, I shall state new achievements and discuss how new functorial decompositions of looped co-H-space arise from an algebraic analysis of functorial coalgebra decompositions of tensor algebras. This is a joint work with Jie Wu.

Compactifications and the Stone spaces of Boolean algebras

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We consider some compactifications of a discrete space, which are the Stone spaces of Boolean algerbas. We examine relations between properties of the algebra and its subsets and properties of the Stone space.

First level Borel isomorphism mapping $L_p(X)$ onto $L_p(Y)$ implies the equality $dimX = dimY$

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Let $L_p(X)$ be the conjugate space for usual continuous function space $C_p(X)$.

A mapping is said to be a first level Borel function, if the inverse image of each F_σ - set is an F_σ - set. The mapping f is called a first level Borel isomorphism if both f and f^{-1} are first level Borel mapping. This notion was introduced in [1].

The following statement is a simultaneous generalization of Jayne-Rogers' theorem [1] and (partially) of Pestov's theorem [2].

Theorem Let X and Y be σ-compact metric spaces and suppose that $L_p(X)$ is a first level Borel isomorphic to $L_p(Y)$. Then $dimX = dimY$.

In our talk we shall also consider some related problems, in particular, a generalization of this theorem on the case of uniform homeomorphisms (in a spirit [3]).

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Cohomology ring of Seifert manifolds. Application to the Borsuk-Ulam theorem

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Joint work of A. Bauval, D. Gonçalves, C. Hayat, P. Zvengrowski.

For a pair (M, τ) where M is a closed manifold and τ is a free involution on M, the Borsuk-Ulam theorem is the answer to the following question: what is the greatest m such that for any continuous map f from M to $\mathbb{R}^m$, there exists $x \in M$ such that $f \circ \tau(x) = f(x)$? These greatest m is called the $\mathbb{Z}_2$-index(M, τ). Our goal is to compute the $\mathbb{Z}_2$-index when M is a 3-dimensional Seifert manifold. The knowledge of the cohomology ring structure of a Seifert manifold is needed to obtain conditions for which $\mathbb{Z}_2$-index$(M, \tau) = 3$.

A separable complete metric n-dimensional space containing isometrically all compact metric n-dimensional spaces

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We shall construct separable complete metric n-dimensional space containing isometrically all compact metric n-dimensional spaces.

Complete pre-isotropic foliations and action-angle variables in contact geometry

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A Liouville integrable Hamiltonian system can be considered as a toric Lagrangian fibration. This approach is reformulated to contact manifolds

$(M, \mathcal{H})$ by Banyaga and Molino. Instead of a toric Lagrangian fibration, one consider an invariant toric fibration transversal to the contact distribution $\mathcal{H}$, such that intersection of tori and $\mathcal{H}$ is a Lagrangian distribution with respect to the conformal class of the symplectic structure on $\mathcal{H}$. Slightly different notion of a contact integrability is given recently by Khesin and Tabachnikov. They defined integrability in terms of the existence of an invariant foliation $\mathcal{F}$, called a co-Legendrian foliation.

Here, based on the Nehoroshev–Mishchenko–Fomenko noncommutative integrability in Hamiltonian mechanics, we introduce an appropriate notion of the noncommutative integrability within a framework of contact geometry.

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Metrizable reminders of locally compact spaces

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This is a joint result with Vitalij Chatyrko. We describe those locally compact noncompact separable metrizable spaces X for which the class

$\mathcal{R}(X)$ of all metrizable remainders of X consists of all metrizable non-empty compacta. We show that for any pair X and X' of locally compact noncompact connected separable metrizable spaces, either $\mathcal{R}(X) \subset \mathcal{R}(X')$ or $\mathcal{R}(X') \subset R(X)$.

Equipartition of several measures

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We prove several results of the following type: any d measures in $\mathbb{R}^d$ can be partitioned simultaneously into k equal parts by a convex partition (this particular result is proved independently by Pablo Soberón). Another example is: Any convex body in the plane can be partitioned into q parts of equal areas and perimeters provided q is a prime power.

The above results give a partial answer to several questions posed by A. Kaneko, M. Kano, R. Nandakumar, N. Ramana Rao, and I. Bárány. The proofs in this paper are inspired by the generalization of the Borsuk–Ulam theorem by M. Gromov and Y. Memarian.

The main topological tool in proving these facts is the lemma about the cohomology of configuration spaces originated in the work of V.A. Vasil'ev.

Algebraic K-theory of stable operator algebras

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Around 1978, it has been conjectured that algebraic and topological K-theories are isomorphic for stable complex C*-algebras. This conjecture has been proved in 1990 by Suslin and Wodzicki, using the notion of H-unital ring due to Wodzicki, with other results due to Cuntz, Higson and Kasparov. In this lecture, we give a new proof of this theorem and extend it to real stable operator algebras. Besides the notion of H-unital ring, we use exact properties of suitable completed tensor product. We also use special Fourier series in order to define Bott elements in algebraic K-theory. This is joint work with Mariusz Wodzicki.

New Polynomial Invariants in Virtual Knot Theory

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In my talk I shall concentrate on some recent developments in constructing invariants of virtual knots that depend on the concept of an affine biquandle. The invariants are simple, powerful and involve a subtle use of parity. The talk will be self-contained.

Topology of algebraically solved systems and Boolean functions

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We suppose that for a completely integrable Hamiltonian system the separation of variables is found in the following form

$$ds_i/d\tau = \sqrt{P_i(s_i, \mathbf{h})}, \quad \mathbf{s} \in \mathbf{R}^k, \quad \mathbf{h} \in \mathbf{R}^m.$$

Here, $\tau(t)$ is some monotonous function, P_i are polynomials in one variable also depending on the vector of arbitrary constants of integration $\mathbf{h}$. If all initial phase variables $\mathbf{x} \in \mathbf{R}^n$ are expressed as rational functions of the radicals of the type $\sqrt{s_i - c_{ij}}$, where c_{ij} are the roots of the polynomials P_i, then we say that the system is algebraically solved.

The examples of such systems are given by the classical solutions in the rigid body dynamics. These are the cases of Euler, Goryachev and Chaplygin, Kovalevskaya for the gravity force field, partial cases of Clebsch, Chaplygin, Goryachev for the motion of a rigid body in fluid etc. Recently, some new cases of the separation of variables with algebraic explicit solution were found for the generalized Kovalevskaya top in a double force field. For these cases the covering of the plane of the separated variables by corresponding integral surfaces in the phase space is of high degree.

Therefore defining the topological type of all integral surfaces becomes technically complicated and tiresome problem.

Given the constants $\mathbf{h}$, the variables $\mathbf{s}$ oscillate in the accessible region $A(\mathbf{h})$. Then to define the topological type of the integral surface we have to explore the multi-valued dependencies $\mathbf{x}(\mathbf{s};\mathbf{h})$ on $A(\mathbf{h})$. It appears that the correspondence between the number of the connected components of the region $A(\mathbf{h})$ and of the integral set $J(\mathbf{h}) \subset \mathbf{R}^n$ can be described in terms of the equivalence classes of Boolean vectors with respect to some Boolean vector functions of special type; we call these Boolean functions algebraical. In fact, these are $\mathbf{Z}_2$-linear mappings of the corresponding vector spaces over $\mathbf{Z}_2$. Thus, the problem of the rough topology investigation for an algebraically solved system is formalized in terms of some invariants of $\mathbf{Z}_2$-linear mappings. We present a number of statements about the linear Boolean vector functions, which allow us to reduce the dimensions of the image and pre-image spaces and calculate the invariants analytically in a simple and clear form. We show various examples including the classical problems and new complicated solutions.

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On the Alexandroff convergence

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In 1948, P.S. Alexandroff introduced a convergence for sequences of continuous functions from a topological space X into a metric space Y and proved that this convergence preserves continuity of the limit function. We consider a statistical version of the Alexandroff convergence and prove that this new kind of convergence is enough for continuity of the limit function. Relations with function spaces and other kinds of convergence will be also discussed.

Normality and Souslin property in Σ-products

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One of the more interesting open problem in the theory of normality in Σ-products is Kodama's question: whether or not every Σ-product of

Lašnev spaces is normal. There is now some model of set theory (due to P.Koszmider) in which Σ-product of Lašnev spaces can be nonnormal.

We give an elementary proof of the following theorem: Let Σ be a Σ-product of Lašnev spaces. If Σ has the Souslin property, then Σ is normal.

Perfect prismatoids and the conjecture concerning with face numbers of centrally symmetric polytopes

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In my talk I shall concentrate on properties of perfect prismatoids in relation to the well-known Kalai conjecture concerning with face numbers of centrally symmetric polytopes .

A centrally symmetric convex polytope P is called a *perfect prismatoid* if $P = conv(F \cup F')$ for any pair of its antipodal facets F and F'.

The Kalai conjecture states that every centrally symmetric d-polytope has at least 3^d non-empty faces. An important class that attains the bound is the class of *Hanner polytopes.* Hanner polytopes are defined recursively: every centrally symmetric 1-polytope is a Hanner polytope. For dimensions $d \geqslant 2$, a d-polytope is a Hanner polytope if it is the direct product or the cross of two lower dimensional Hanner polytopes.

The aim of my talk is to prove that any Hanner polytope is a perfect prismatoid and construct a 5-dimensional perfect prismatoid that is not a Hanner polytope. In dimensions $d < 5$ any perfect prismatoid is a Hanner polytope and vice versa.

Also it will be proved that any d-dimensional perfect prismatoid is affine equivalent to some 0/1-polytope that is the convex hull of a subset of the point set $\{0, 1\}^d$.

Symplectic invariants of almost toric 4-manifolds

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Almost toric manifolds form a class of singular Lagrangian fibered symplectic manifolds that naturally generalize toric manifolds, Lagrangian bundles and momentum maps with nondegenerate nonhyperbolic singularities from the theory of integrable Hamiltonian systems. In dimension four almost toric manifolds also can be described as Lagrangian fibrations with either elliptic or focus-focus singular points.

Almost toric manifolds were introduced by Margaret Symington in [1] and in dimension four they were classified up to diffeomorphism by Naichung Conan Leung and Margaret Symington in [2].

In the talk we will describe different invariants of almost toric 4-manifolds and explain their classification up to fiberwise symplectomorphism.

The case when there is no singular points was completely studied in [3] and can also be found in [4, 5].

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Strongly locally homogeneous spaces and their completions

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Strongly locally homogeneous (SLH) spaces were introduced by L. Ford, who proved that they are coset spaces. For metrizable spaces J. van Mill sharpened this result showing that separable metrizable (Polish) SLH space is a coset space of a separable metrizable (Polish) group. We give characterization of Polish SLH spaces and show that every separable metrizable SLH space has a completion which is a Polish SLH space. Moreover this completion is obtained in agreement with the completion in two-sided uniformity of the group that realizes the SLH property of a SLH space.

Conjugation invariants of area-preserving self-diffeomorphisms of a 2-disk

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Denote by $S = \mathrm{Symp}_c(D)$ the group of area-preserving self-diffeomorphisms of a 2-disk $D = D^2$ that are identical on the boundary of the disk. A function $I : S \to \mathbb{R}$ such that $I(ghg^{-1}) = I(h)$ for all $g, h \in S$ is called a *conjugation invariant* on the group S.

An example of a conjugation invariant is the *Calabi invariant* (1970), $\mathrm{Cal}(h) := \int_0^1 \int_D H_t(p,q) dp\, dq\, dt$, $h \in S$. Here $g_t \in S$ is a path in the group S joining $h = g_1$ to the identity $g_0 = \mathrm{id}_D$, while the function $H_t = H_t(p,q)$ on the disk D is defined by the equalities $dg_t(p,q)/dt = (-\partial H_t(g_t(p,q))/\partial q,\, \partial H_t(g_t(p,q))/\partial p)$ and $H_t|_{\partial D} = 0$. Such a path g_t (together with a family of functions H_t) exists for any $h \in S$, since S is path-connected with respect to C^∞-topology. The Calabi invariant is a differentiable homomorphism, moreover its differential at any element $h \in S$ has the form $dI(h)(H) = \int_D H(p,q) dp\, dq$ (and, hence, does not depend on h). Here the tangent space at h to S is identified with the space of all Hamiltonian vector fields $v = (-\partial H/\partial q, \partial H/\partial p)$ on D, where H is a smooth function on D vanishing at ∂D. A. Banyaga (1978) proved that any homomorphism $I : S \to \mathbb{R}$ has the form $I = A\circ \mathrm{Cal}$ for a suitable automorphism A of the group $(\mathbb{R}, +)$.

Other examples of conjugation invariants on the group S are the area of the fixed point set $\mathrm{Fix}(h)$, the Hofer norm $\|h\|_\infty = \inf_{\{g_t\}} \int_0^1 (\max_D |H_t|)dt$ due to H. Hofer (1993), and certain "spectral invariants" [P]. These invariants are not differentiable, but are Lipschitz only.

We prove that any differentiable conjugation invariant I on S has the form $I = A\circ \mathrm{Cal}$ for a suitable function $A : \mathbb{R} \to \mathbb{R}$, provided that $dI(h)(H) = \int_D K(h,p,q)H(p,q)dp\,dq$ for some continuous function $K : S \times D \to \mathbb{R}$ with respect to C^1-topology on S.

This result can be generalized to the group $\mathrm{Ham}_c(M,\omega)$ of compactly supported Hamiltonian self-maps of an open surface M equipped with an area form ω. Our proof uses the result by Ch. Bonatti and S. Crovisier [BC] that the C^1-generic volume-preserving diffeomorphism $h \in S$ admits a dense orbit. Our result seems to be related to the result by D. Serre [S] about the first order invariants of divergence free vector fields on $\mathbb{R}^3$.

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On multiplicative functionals on the space of continuous functions

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Let X be a Tychonoff space and $C_p(X)$ be a space of all continuous real-valued functions on X, endowed with the topology of pointwise convergence.

Definition 1. A mapping $x_1^{p_1} \cdot \ldots \cdot x_k^{p_k} : C_p(X) \to R$ will be said to be a **Monomial**, if it defined by the rule

$$x_1^{p_1} \cdot \ldots \cdot x_k^{p_k}(\varphi) = (\varphi(x_1))^{p_1} \cdot \ldots \cdot (\varphi(x_k))^{p_k},$$

where $\varphi \in C_p(X)$ and $x_i \in X$, $p_i \in N$ for every $i \in N$.

The subspace in $C_pC_p(X)$, consisting of all monomials we denote by $D_p(X)$.

Theorem 1. Every multiplicative functional on $C_p(X)$ is a monomial.

Definition 2. Let $K : D_p(X) \to X$ be a finite-valued mapping, defined by the rule $K(f) = \{x_1, \cdots, x_k\}$ for each monomial $f = x_1^{p_1} \cdot \ldots \cdot x_k^{p_k}$. This mapping will be called by the **support-mapping**.

Theorem 2. The support-mapping K is surjective and upper semicontinuous.

Theorem 3. Let $h : C_p(X) \to C_p(Y)$ be a homeomorphism 'onto' and $h^* : C_pC_p(Y) \to C_pC_p(X)$ be its natural conjugate mapping. If $h^*(Y) \subset D_p(X)$ and $(h^*)^{-1}(X) \subset D_p(Y)$. then

(a) X is compact iff Y is compact;

(b) $l(X) = l(Y)$.

On the stability of the colored Jones polynomial

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We prove the stability of the coefficients of the colored Jones polynomial of an alternating link and present a generalized Nahm sum formula for the resulting power series, defined in terms of a reduced diagram of the alternating link. This is joint work with S. Garoufalidis.

Nonautonomous flows and uniform topology

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Let M be a C^∞-smooth closed manifold, $\mathcal{V}^r(M)$ be the Banach space of C^r-smooth vector fields on M endowed with C^r-norm. A C^r-smooth *nonautonomous vector field* on M is an uniformly continuous bounded map $v : \mathbb{R} \to \mathcal{V}^r(M)$. If this map v is also uniformly continuous C^s-differentiable map, we call v to be a $C^{r,s}-$smooth nonautonomous vector field. A nonautonomous vector field v (below NVF, for brevity) generates a foliation F of manifold $M \times \mathbb{R}$ into its integral curves $\{(x(t), t)\}$. Henceforth we consider $M \times \mathbb{R}$ with its standard uniform structure. All uniformly continuous maps of $M \times \mathbb{R}$ to itself are considered with respect to this uniform structure. A notion of equivalent nonautonomous vector fields proposed in [1] is as follows.

Definition. *Two NVFs* v_1, v_1 *are uniformly equivalent if foliations* F_1, F_2 *are uniformly equivalent, i.e. there is an equimorphism (uniform homeomorphism having a uniform inverse homeomorphism)* $h : M \times \mathbb{R} \to M \times \mathbb{R}$ *respecting foliations (i.e. sending an every integral curve* γ *of* F_1 *to an integral curve of* F_2 *along with its orientation in* $\mathbb{R}$*).*

It is clear that this equivalency relation distinguishes NFVs in which the asymptotic behavior of integral curves are different. This relation allows us to introduce the notion of the structurally stable NVFs.

Definition. *An NVF* v *is structurally stable if there is a neighborhood of* v *in the space of* $NVFs$ *such that all NVFs in this neighborhood are uniformly equivalent.*

The development of the modern theory of (autonomous) dynamical system showed that this equivalency relation is too rigid and well suited for the classification of systems with a simple structure, now known as Morse-Smale systems (flows and diffeomorphisms). In analogy, for nonautonomous vector fields a class of Morse-Smale type vector fields was distinguished (L.Lerman). For such NVFs it was shown the Morse type inequalities to be valid. For gradient NVFs on two-dimensional closed manifolds a complete invariant of the uniform equivalency was found, like the Leontovich's scheme on S^2 or Peixoto's graph on M^2 for autonomous vector fields.

Now suppose a NVF v to be of the gradient type and almost periodic in time, that is the map $v : \mathbb{R} \to \mathcal{V}^r(M)$ is almost periodic.

Theorem. Every integral curve of v tend as $t \to \infty$ to some almost periodic integral curve Γ_1 and to another almost periodic integral curve Γ_2 as $t \to -\infty$. Both $\Gamma_{1,2}$ possess exponential dichotomy of $\mathbb{R}$. The number of these Γ_i is finite and its types obey to the Morse inequalities.

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Orbit configuration spaces of small covers and quasi-toric manifolds

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In this talk, we investigate the orbit configuration spaces of some equivariant closed manifolds over simple convex polytopes in toric topology, such as small covers, quasi-toric manifolds and (real) moment-angle manifolds; especially for the cases of small covers and quasi-toric manifolds. These kinds of orbit configuration spaces are all non-free and noncompact, but still built via simple convex polytopes. We obtain an explicit formula of Euler characteristic for orbit configuration spaces of small covers and quasi-toric manifolds in terms of the h-vector of a simple convex polytope. As a by-product of our method, we also obtain a formula of Euler characteristic for the classical configuration space, which generalizes the Félix-Thomas formula. In addition, we also study the homotopy type of such orbit configuration spaces. In particular, we determine an equivariant strong deformation retract of the orbit configuration space of 2 distinct orbit-points in a small cover or a quasi-toric manifold, which turns out that we are able to further study the Betti numbers and (equivariant) cohomology of such an orbit configuration space. This is a joint work with Junda Chen and Jie Wu.

Convex Hull of a Poisson Point Process in the Clifford Torus

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In the 4-dimensional Euclidean space $\mathbb{E}^4$ consider the two-dimensional Clifford torus

$$T^2 = \{(\cos\phi, \sin\phi, \cos\psi, \sin\psi) : -\pi < \phi, \psi \leq \pi\}.$$

Clearly, T^2 is a submanifold of the three-dimensional sphere

$$S^3_{\sqrt{2}} = \{(\xi_1, \xi_2, \xi_3, \xi_4) : \xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2 = 2\}.$$

N. Dolbilin and M. Tanemura [1] performed a computer simulation of a Poisson process $P_\lambda(\omega) \subset T^2$. As a result, certain conjectures on the combinatorics of a random polytope $\Pi(\lambda) = conv P_\lambda(\omega)$ have been posed. In particular, it was suggested that the mean valence of a vertex of $\Pi(\lambda)$ has expectation of magnitude $O^*(\ln\lambda)$.

The following results hold for $\Pi(\lambda)$.

Theorem 2 *For* $i = 1, 2, 3$ *one has* $\mathsf{E}\, f_i(\Pi(\lambda)) = O^*(\lambda \ln \lambda)$ *as* $\lambda \to \infty$.

Theorem 3 *The expectation of the mean valence of a vertex of* $\Pi(\lambda)$ *has asymptotics* $\mathsf{E}\, \bar{v}(\Pi(\lambda)) = O^*(\ln\lambda)$ *as* $\lambda \to \infty$.

Remark 1 *The statement of theorem 3 is exactly the conjecture posed by Dolbilin and Tanemura.*

One possible way to prove theorems 2 and 3 involves the *cap covering technique* described in [2]. The same technique allows to estimate the variance of $f_3(P(\lambda))$.

Theorem 4 $var f_3(\Pi(\lambda)) \ll \lambda \ln^2 \lambda$.

Theorem 4 immediately implies *the law of large numbers* for $f_3(P(\lambda))$.

Theorem 5

$$\mathsf{P}\left(\left|\frac{f_3(\Pi(\lambda)) - \mathsf{E}\, f_3(\Pi(\lambda))}{\mathsf{E}\, f_3(\Pi(\lambda))}\right| > \varepsilon\right) \ll \frac{1}{\lambda\varepsilon^2}.$$

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Affine reducibility

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We consider families of 0/1-polytopes related to well-known NP-complete problems: knapsack, traveling salesman, graph colouring, independent set, 3-assignment, set covering, 3-SAT, and some others. We propose a way of structuring the set of combinatorial polytopes. Its basis is the classical notion of polynomial reducibility. It turns out that if some combinatorial problem X reduces to some problem Y then the polytope of X is usually an affine image of a face of the polytope of Y. In some cases the appropriate affine transformation is invertible, i.e. the polytope of X is affinely equivalent to a face of the polytope of Y. Hence, the polytope of X has a more simple structure then the polytope of Y. We show that boolean quadratic polytopes are faces of the mentioned 0/1-polytopes.

Random knots

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In my talk I shall discuss methods for generating a random knot. A random knot can be obtained from tilings in the Euclidean and hyperbolic planes, via random walks on the braid group or the mapping class groups of closed surfaces, with the help of Poisson distributions, etc. A random knot is knotted with high probability, under almost all reasonable approaches. An interesting fact is that some methods of generating a knot produce mostly composite knots, while under some other approaches prime knots prevail drastically. This problem (prime/composite) is still open for many methods. Questions emerging in this area of research are related to random walk theory, hyperbolic geometry, percolation theory, etc.

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Prime Decompositions and the Diamond Lemma

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We develop a new version of the famous Diamond Lemma and describe several results on decompositions of different geometric objects. All results are obtained by using that version.

1. The Kneser-Milnor and Swarup prime decomposition theorems of 3-manifolds into connected sums and boundary connected sums (new proofs).
2. A prime decomposition theorem for knotted graphs in 3-manifolds containing no non-separating 2-spheres.
3. Disproving the "folklore" prime decomposition theorem for 3-orbifolds.
4. A new theorem on annular splittings of 3-manifolds, which is independent of the JSJ-decomposition theorem.
5. The prime decomposition theorem for homologically trivial knots in direct products of surfaces and intervals.
6. The prime decomposition theorem for virtual knots.

Metric h-homogeneous spaces

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I shall survey some recent results on metric h-homogeneous spaces.

A metric space X is called *h-homogeneous* if every nonempty clopen subset of X is homeomorphic to X and $\mathrm{Ind}X = 0$.

Among homogeneous spaces, h-homogeneous ones often have additional properties. We pick out some examples of such properties. The notion of an h-homogeneous space was firstly applied to the Borel sets. The first significant results about metric h-homogeneous spaces were obtained by

A.V. Ostrovsky, J. van Mill, and F. van Engelen. h-Homogeneity has recently been studied in the case of compact or Tychonoff spaces.

Theorem 1. Let X and Y be metric h-homogeneous spaces of first category. Suppose X is homeomorphic to an F_σ-subset of Y and Y is homeomorphic to an F_σ-subset of X. Then X is homeomorphic to Y.

Theorem 2. Let X^ω be a metric space of first category, $\mathrm{Ind}X = 0$, and F be an F_σ-subset of X^ω. Then $F \times X^\omega$ is homeomorphic to X^ω.

Theorem 3. Let metric spaces X and Y have dense topologically complete subspaces and $\mathrm{Ind}X = \mathrm{Ind}Y = 0$. If every nonempty open subset of X contains a closed copy of Y and every nonempty open subset of Y contains a closed copy of X, then X is homeomorphic to Y.

Theorem 4. Let X be a metric homogeneous space of weight k such that $\mathrm{Ind}X = 0$, $w(U) = k$ for every nonempty open set $U \subset X$, and $\mathrm{cf}(k) > \omega$. Then X is an h-homogeneous space.

Theorem 5. Let X be a separable zero-dimensional metrizable topological group which is not locally precompact. Then X is an h-homogeneous space.

Combinatorics of collapsible polyhedra and maps

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The aim of this talk is to give two combinatorial characterizations of collapsible polyhedra: (i) in terms of zipping of posets, as defined by N. Reading [Electron. J. Combin., 11 (2004), R#74], and (ii) in terms of constructible posets, which generalize *acyclic* constructible simplicial complexes in the sense of Topological Combinatorics, originating in M. Hochster's work on Cohen–Macalay rings [Ann. Math. 96 (1972), 318–337].

A simplicial complex, and more generally a cell complex (=finite CW-complex whose attaching maps are PL embeddings) can be reconstructed from, and so will be identified with, the poset of its *nonempty* faces.

Let P be a poset. If a $p \in P$ covers two incomparable elements $q, r \in P$, and every $s < p$ with $s \neq q, r$ satisfies $s < q$ and $s < r$, then P is said to *elementarily zip* onto the quotient (in the concrete category of posets over the category of sets) of P by the subposet $(\{p, q, r\}, \leq)$. A *zipping* of posets is a sequence of elementary zippings.

Theorem 1. *Let X be a polyhedron. The following are equivalent:*
(i) X is collapsible;
(ii) X can be triangulated by a simplicial complex that zips onto a point;
(iii) X can be cellulated by a cell complex that zips onto a point.

We call a poset P *constructible* if either P has a greatest element or $P = Q \cup R$, where Q and R are order ideals, each of Q, R and $Q \cap R$ is constructible, and every maximal element of $Q \cap R$ is covered by a maximal element of Q and by a maximal element of R, and not covered by non-maximal elements of P. Order complexes of constructible posets are clearly contractible. No triangulation of the dunce hat is constructible, but there exists a constructible 2-dimensional simplicial complex whose underlying polyhedron is not collapsible [M. Hachimori, Discrete Math., 308 (2008), 2307–2312].

Theorem 2. *A cell complex K zips onto a point if and only if the dual poset K^* is constructible.*

A PL map is called *collapsible* if its point-inverses are collapsible (so in particular nonempty). By a well-known result of M. M. Cohen, composition of collapsible retractions is a collapsible retraction [Trans. Amer. Math. Soc. 136 (1969), 189–229].

Corollary. *Composition of collapsible maps is collapsible.*

The only proof known to the speaker relies on Theorems 1 and 2.

Self-adjoint commuting ordinary differential operators

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We find sufficient conditions when an operator of fourth order commuting with an operator of order $4g + 2$ is self-adjoint. We introduce an equation on potentials $V(x), W(x)$ of the self-adjoint operator $L = (\partial_x^2+V)^2+W$ and some additional data. With the help of this equation we find the first example of commuting differential operators of rank two corresponding to a spectral curve of arbitrary genus. These operators have polynomial coefficients and define commutative subalgebras of the first Weyl algebra.

Construction of classifying space of transitive Lie algebroids

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Transitive Lie algebroids have specific properties that allow to look at the transitive Lie algebroid as an element of the object of a homotopy functor. Roughly speaking each transitive Lie algebroids can be described as a vector bundle over the tangent bundle of the manifold which is endowed with additional structures. Therefore transitive Lie algebroids admits a construction of inverse image generated by a smooth mapping of smooth manifolds.

Due to to K.Mackenzie ([1]) the construction can be managed as a homotopy functor $TLA_{\mathfrak{g}}$ from category of smooth manifolds to the transitive Lie algebroids. The functor $TLA_{\mathfrak{g}}$ associates with each smooth manifold M the set $TLA_{\mathfrak{g}}(M)$ of all transitive algebroids with fixed structural finite dimensional Lie algebra $\mathfrak{g}$. Hence one can construct ([2],[3]) a classifying space $\mathcal{B}_{\mathfrak{g}}$ such that the family of all transitive Lie algebroids with fixed Lie algebra $\mathfrak{g}$ over the manifold M has one-to-one correspondence with the family of homotopy classes of continuous maps $[M, \mathcal{B}_{\mathfrak{g}}]$: $\mathcal{A}(\mathcal{M}) \approx [\mathcal{M}, \mathcal{B}_{\mathfrak{g}}]$.

In spite of the evident categorical point of view we faced the challenge of geometrical construction of the classifying space, in particular generalization of the Eilenberg-MacLane spaces, realization of the cohomological obstructions for equivariant mapping and others.

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Non-commutative signature and fixed points

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In my talk will be shown how to reduce the computation of the non-commutative signature to its computation on fixed points sets. This will be done in terms of bordisms of algebraic Poincaré complexes.

On a Boundary of a Neighborhood of an Isolated Stationary Point of a Planar Conical Local Dynamical System

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We consider the structure of a neighborhood of an isolated stationary point (ISP) of a planar conical local dynamical system (LDS). We study the structure of a neighborhood of an ISP usinig the axiomatic theory of ordinary differential equations (see Filippov V.V., Solution Spaces of Ordinary Differential Equations, Moscow: Izd. Moskov. Univ., 1993). The main result is the following

Theorem. A neighborhood of an ISP of a planar conical LDS is regular (see Mychka E.Yu., On the Structure of a Neighborhood of an Isolated Stationary Point of a Local Dynamical System on a Plane, Differ. Uravn., 2011, vol. 47, no. 2, pp. 195-208).

To obtain the main result we construct a neighborhood with the special boundary. In addition to transversal properties the boundary meet each ray only at one point.

On three functors: β, λ and υ

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The talk relates joint work with Themba Dube on the Stone-Čech compactification βL, the Lindelöf coreflection λL and the Realcompact coreflection υL of a completely regular frame L. In this pointfree setting, the classical notions of compactness, Lindelöfness and paracompactness have their traditional characterizations via covers. In this talk we consider special commutative diagrams which we call *round squares* and show that the above classical notions, inclusive of realcompactness, may be characterized in terms of these type of diagrams.

On Some Property of Axial Diameters of a Simplex

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Let C be a convex body in $\mathbb{R}^n$. Denote by σC the homothetic copy of C with center of homothety in the center of gravity of C and coefficient σ. By $\alpha(C)$ denote the minimal $\sigma > 0$ such that $Q_n = [0,1]^n$ is contained in a translate of σC. Let $d_i(C)$ be the ith axial diameter of C, i.e., the length of a longest segment in C parallel to the ith coordinate axis. The main result of my talk is the following statement.

Theorem. *For each C,*

$$\alpha(C) \leq \sum_{i=1}^{n} \frac{1}{d_i(C)}. \tag{1}$$

If S is a nondegenerate simplex, then

$$\alpha(S) = \sum_{i=1}^{n} \frac{1}{d_i(S)}.$$

In other words, in the case $C = S$ there is an equality in (1).

I also mention some corollaries of this theorem.

Framings of knotted graphs

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The main aim of the talk is to define an analogue of the framing of a classical knot for knotted graphs and to classify the framings of a knotted graph up to isotopy.

Geometry of integrable non-Hamiltonian systems

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I will present a geometric theory of integrable dynamical systems, with results about normal forms and geometric linearization of singularities, local, semi-local and global invariants and classification, obstructions to integrability, and related geometric objects, including generalized toric manifolds, commuting foliations, etc.

Topology of the Liouville foliation in the integrable case of Goryachev in the problem on motion of a rigid body in fluid

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The generalized Kirchhoff equations of rigid body motion in fluid have the form

$$\dot{s} = s \times \frac{\partial H}{\partial s} + r \times \frac{\partial H}{\partial r}, \;\; \dot{r} = r \times \frac{\partial H}{\partial s}, \tag{1}$$

[4]The research was supported by the RFBR (grant 10-01-00748), the Programme for Support of Leading Scientific Schools (grant NSh-3224.2010.1), the Programme for Development of Scientific Potential of Higher Education (grant RNP-2.1.1.3704), and the federal target programme "Scientific and Scientific-Pedagogical Personnels of Innovative Russia" (grant nos. 02.740.11.5213, 14.740.11.0794).

where $s, r \in \mathbb{R}^3$ are the impulse moment and the impulse force respectively, $H = H(s, r)$ is the total energy. This system of equations always possesses the geometric integral $f_1 = r_1^2 + r_2^2 + r_3^2$, the area integral $f_2 = s_1r_1 + s_2r_2 + s_3r_3$, and the energy integral H. At the common level set $\{f_1 = a^2,\ f_2 = g\}$ the system is Hamiltonian. In [1] D. N. Goryachev found an integrable case where

$$H = \frac{1}{2}(s_1^2 + s_2^2 + 2s_3^2) + \frac{1}{2}c(r_1^2 - r_2^2) + \frac{b}{2r_3^2}.$$

In [2], on the basis of Boolean functions method of M. P. Kharlamov [3], P. E. Ryabov obtained the real seraration of variables for the Goryachev case which allowed to study phase topology of the system.

For the partial case $b = 0$ integrability of the system (1) was proved by S. A. Chaplygin in [4]. In the case $g = 0$ he found an additional integral and also reduced the problem to elliptic quadratures. In [5] topology of the Liouville foliation in the case $b = 0$ was investigated (topological type of energy surfaces, bifurcation sets, bifurcations of Liouville tori). In the present talk for the Chaplygin case we calculate the Fomenko-Zieschang invariant which is known to be a complete invariant for the Liouville equivalence (see [2]).

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Hypergraphs of a special type and properties of the cut polytope relaxations

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Consider the set of 3-uniform mixed hypergraphs of the form $G = (V, E, A)$, where V is the vertex set, i.e., $V = \mathbb{N}_n = \{1, 2, 3, \ldots, n\}$; E is the set of unoriented edges, i.e., $E = \{(i, j, k)\} \subseteq \mathbb{N} \times \mathbb{N} \times \mathbb{N}$; and A is the set of oriented edges, i.e., $A = \{((i, j), k)\} \subseteq \mathbb{N} \times \mathbb{N} \times \mathbb{N}$, where the pair of vertices (i, j) is the beginning of the edge and the vertex k is the end of the edge.

We introduce the operation of inversion of the i-th vertex in a hypergraph $G = (V, E, A)$ which transforms all edges incident to this vertex, as follows:

$$(i, j, k) \to ((j, k), i),$$
$$((j, k), i) \to (i, j, k),$$
$$((i, j), k) \to ((i, k), j).$$

Let G_I denote the class of hypergraphs $G = (V, E, A)$ for which the set E of unoriented edges is nonempty and remains nonempty under all possible inversions.

We use hypergraphs of the form specified above to describe properties of the cut polytope relaxations.

The object of the research is the class of rooted semimetric polytopes M_n. Polytopes from this class have a number of special features, which provoke significant interest in such polytopes.

The polytope M_n^Z, generated by all integer vertices of M_n is called the cut polytope, because the well-known NP-complete problem of maximal cut reduces to optimizing a linear objective function on M_n^Z. Therefore, M_n is a relaxation polytope for the cut problem.

Let us define following sequence of higher level nested relaxations of the cut polytope, described by a special procedure:

$$CUT(n) = M_{n,n} \subseteq M_{n,n-1} \subseteq ... \subseteq M_{n,4} \subseteq M_{n,3} \subseteq M_{n,2} = M_{n,1} = M_n.$$

Note that each point $u \in M_{n,3}$ can be assigned a 3-uniform mixed hypergraph of the form specified above in accordance with certain rules, which we call the hypergraph of the point $G(u)$.

Theorem 1. *If the hypergraph $G(u)$ of some point $u \in M_{n,3}$ belongs to the class G_I, then any decomposition of u in a convex combination of vertices of $M_{n,3}$ contains no integer vertices.*

Based on the Theorem 1 was proved the following

Theorem 2. *For any $n \geq 5$ and $q \geq 195$, there exist points u in the polytope $M_{n,4}$ and v in the polytope $M_{q,5}$ whose hypergraphs $G(u)$ and $G(v)$ belong to the class G_I.*

Isometrical embeddings of finite metric spaces

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We shall concentrate on some problems on isometrical embeddings. Let $\mathbb{F}_d^n$ be the class of all finite metric spaces with diameter $\leqslant d$ consisting of less than or equal to n points. It will be proved that for every $n \in \mathbb{N}$ there is a metric on the Cantor set such that every element of $\mathbb{F}_1^n$ isometricaly embedds into this Cantor set.

On piecewise smooth cohomology of locally trivial Lie groupoids

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A. Mishchenko and J. Oliveira in [3] defined the notion of piecewise smooth cohomology of transitive Lie algebroids defined over combinatorial manifolds and proved that piecewise smooth and Lie algebroid cohomology of a transitive Lie algebroid over a combinatorial manifold are isomorphic. In this talk, we describe an application of that result which consists in the relationship between piecewise de Rham cohomology of left invariants forms of a locally trivial Lie groupoid on a combinatorial manifold and piecewise smooth cohomology of its Lie algebroid.

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On σ-countably-compact of space $C_\lambda(X)$

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The authors offer a characterization of the P-property of topological space X in terms of space $C_\lambda(X)$ of continuous real functions on Tychonoff space X in the set-open topology. It is shown that space $C_\lambda(X)$ of continuous real functions on X is σ-countably-compact in the set-open topology if and only if

1. X is a pseudocompact space;
2. set $X(P)$ of P-points of X is dense in X;
3. the family λ consists of finite subsets $X(P)$;
4. $C_p(X(P)|X)$ is σ-countably-compact.

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On uniform Eberlein compacta

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Recall that a compactum is called *uniformly Eberlein* ($\equiv$ *uE*) if it is homeomorphic to a subspace of a Hilbert space in its weak topology and a compactum X is *uE* iff there exists a T_0-separating X functionally open family λ in X such that it is the union of families λ_i of order $\leq n(i) \in \mathbb{N}_0 = \{0, 1, 2, \dots\}$ and $i \in \mathbb{N}$.

A compactum X will be called *n-uniformly Eberlein* ($\equiv$ *n-uE*) if there exists a T_0-separating X functionally open family λ in X such that it is the union of families λ_i of order $\leq n \in \mathbb{N}_0$ and $i \in \mathbb{N}$.

Theorem. *For any $n \in \mathbb{N}_0$, there exists an n-uE compactum $X_n \neq \emptyset$ such that for $n \in \mathbb{N}$, X_n is not (n-1)-uE compactum. The Alexandroff ($\equiv$ one-point) compactification X_∞ of the discrete union of all X_n is an uE compactum that is not n-uE compactum for all $n \in \mathbb{N}_0$. All X_n, $n \in \mathbb{N}_0 \cup \{\infty\}$, are not metrcompacta.*

The class of all n-uE compacta is countably productive. There exists a universal element in the class of all n-uE compacta of weight $\leq \tau$, $n \in \mathbb{N}_0$, $\tau \geq \omega$.

On Volodin space for Bruns–Gubeladze K-theory

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In series of papers W.Bruns and J.Gubeladze had generalized some constructions from classical algebraic K–theory. One can observe that relations like $[e^a_{ij}, e^b_{jk}] = e^{ab}_{ik}$ in the group of elementary matrices can be encoded not in terms of pairs of indices ij but in terms of vectors $e_i - e_j$ where e_i and e_j are vectors corresponding to vertices of simplex.

Encoding the relations of such type by distinguished collection of vectors vectors is possible for the class class of so called balanced polytopes. Corresponding vectors are called column vectors. Bruns and Gubeladze had generalized group of elementary martices for balanced polytopes $E(A, P)$

and defined analog of Quillen K–theory by $K_i^Q(A,P) = \pi_i(BE(A,P)^+)$, where $i \geq 2$, A is an assocaitive ring with unit, P is a balances polytope.

Also they had defined two different versions of Volodin space, and prove that for so called Col-divisible polytopes two versions of Volodin space are homotopy equivalent, Volodin K–theory $K_i^V(A,P)$ is defined and coincides with $K_i^Q(A,P)$.

In our talk we shall consider examples of non-Col-divisible polytopes and prove that for some of them one version of Volodin space gives us K-theory which is equivalent to Quillen K-theory.

Bypasses for rectangular diagrams.

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In my talk I will discuss a criteria for a rectangular diagram to admit a simplification which is given in terms of Legendrian knots. This criteria is introduced in joint work with I. Dynnikov. This criteria provides that there are two types of simplifications which are mutually independent in a sense. As a consequence we have: a new proof of the monotonic simplification theorem for the unknot; that a minimal rectangular diagram maximizes the Thurston–Bennequin number for the corresponding links; and a proof of Jones' conjecture about the invariance of the algebraic number of intersections of a minimal braid representing a fixed link type.

Polytopes and K-theory

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At the end of 1990s Winfried Bruns and Joseph Gubeladze introduced a new subclass of lattice polytopes called *balanced*. For such polytopes and associative, commutative ring R with unity they generalised the group of elementary matrices to the group of elementary automorphisms of polytopal algebra which is a subgroup of the group of graded automorphisms of polytopal algebra. Starting with this group one can “polyhedrize” algebraic K-groups and obtain new K-groups which depend on balanced polytope P and ring R mentioned above. We will denote such groups $K_i(R,P)$. Since we start with elementary matrices and do not have “good” analogue for $GL_n(R)$ there is no $K_0(R,P)$ and $K_1(R,P)$. On the other hand Bruns and Gubeladze proved that a lot of properties of $K_2(R)$ take place for $K_2(R,P)$. Here is the list of some of such properties and new results specific for polyhedral K-theory:

1. If P is a standard simplex (of arbitrary dimension) $K_2(R,P) \cong K_2(R)$.
2. For any balanced P there exist a universal central extension of polytopal groups
$$1 \to K_2(R,P) \to St(R,P) \to E(R,P) \to 1.$$
3. For some balanced polytopes $K_2(R,P)$ is not isomorphic to $K_2(R)$. First examples are availible in dimension 2 (for balanced polygons). But if R has many units one of the following take place (in dimension 2):
$$K_2(R,P) \cong K_2(R) \quad \text{or} \quad K_2(R,P) \cong K_2(R) \oplus K_2(R).$$

The open question is: “Are there any balanced polytopes which gives us new K-groups, not just a sum of ordinary ones?” Bruns and Gubeladze proposed that a good candidate for such polytope is a pyramid over the unit square.

In my talk I'm going to explain why this example doesn't work (it gives us a sum of 3 ordinary K-groups) and how one can calculate K_2 for arbitrary 3-dimensional balanced polytope.

Uniformly continuous and slowly oscillating functions on metric spaces

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The metrics d, ρ on a set X is said to be *uniformly equivalent* if, for every $\varepsilon > 0$, there exists $\delta > 0$ such that, for all $x, y \in X$,

$$d(x, y) < \delta \Rightarrow \rho(x, y) < \varepsilon, \ \ \rho(x, y) < \delta \Rightarrow d(x, y) < \varepsilon.$$

A subset A of (X, d) is *uniformly discrete* if there exists $\varepsilon > 0$ such that $d(x, y) > \varepsilon$ for all distinct $x, y \in A$. We denote by $UC(X, d)$ and $UD(X, d)$ the families of all bounded uniformly continuous functions and uniformly discrete subsets of (X, d).

Theorem 6 *For the metrics d, ρ on a set X, the following statements are equivalent:*

(i) d, ρ are uniformly equivalent;

(ii) $UC(X, d) = UC(X, \rho)$;

(iii) $UD(X, d) = UD(X, \rho)$.

The metrics d, ρ on a set X are said to be *asymptotically equivalent* if, for every $\varepsilon > 0$, there exists $\delta > 0$ such that, for all $x, y \in X$,

$$d(x, y) < \varepsilon \Rightarrow \rho(x, y) < \delta, \ \ \rho(x, y) < \varepsilon \Rightarrow d(x, y) < \delta.$$

A subset A of (X, d) is *thin* if, for every $\varepsilon > 0$, there exists a bounded subset V of X such that $d(x, y) > \varepsilon$ for all distinct $x, y \in A \setminus V$ (V is *bounded* if it is contained in some ball). We denote by $Bound(X, d)$ and $Th(X, d)$ the families of all bounded and thin subsets of (X, d).

A function $f : (X, d)\ to \mathbb{R}$ is called *slowly oscillating* if, for all $\varepsilon > 0$, $\delta > 0$, there exists a bounded subset V of (X, d) such that, for all $x, y \in X \setminus V$, $d(x, y) < \delta \Rightarrow |f(x) - f(y)| < \varepsilon$. We denote by $SO(X, d)$ the family of all bounded slowly oscillating functions on (X, d).

Theorem 7 *For the metrics d, ρ on a set X with $Bound(X, d) = Bound(X, \rho)$, the following statements are equivalent:*

(i) d, ρ are asymptotically equivalent;

(ii) $SO(X,d) = SO(X,\rho)$*;*

(iii) $Th(X,d) = Th(X,\rho)$*.*

Distributivity versus associativity: homology theory applied to knot theory

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While homology theory of associative structures, such as groups and rings, has been extensively studied in the past beginning with the work of Hopf, Eilenberg, and Hochschild, homology of non-associative distributive structures, such as quandles, were neglected until recently. Distributive structures have been studied for a long time. In 1880, C.S. Peirce emphasized the importance of (right) self-distributivity in algebraic structures. However, homology for these universal algebras was introduced only seventeen years ago by Fenn, Rourke, and Sanderson. We develop this theory in the historical context and propose a general framework to study homology of distributive structures. We illustrate the theory by computing some examples of 1-term homology (in particular showing nontrivial torsion part), and then discussing 4-term homology for Boolean algebras. We outline potential relations to Khovanov homology of links, via the Yang-Baxter operator.

$\mathbf{F_q[M_n]}$, $\mathbf{F_q[GL_n]}$ and $\mathbf{F_q[S_n]}$ as Quantized Universal Enveloping Algebras

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After introductory part to the problem which we studied, in the second part we considered the quantum function algebra $F_q[GL_n]$, and study the subset

$$\mathcal{F}_q[GL_n] \;:=\; \left\{ f \in F_q[GL_n] \;\middle|\; \left\langle f, \mathcal{U}_q(\mathfrak{gl}_n) \right\rangle \subseteq \mathbb{Z}\big[q, q^{-1}\big] \right\} ,$$

of all elements of $F_q[GL_n]$ which are $\mathbb{Z}\big[q, q^{-1}\big]$–valued when paired with $\mathcal{U}_q(\mathfrak{gl}_n)$, the unrestricted $\mathbb{Z}\big[q, q^{-1}\big]$–integral form of $U_q(\mathfrak{gl}_n)$ introduced by

De Concini, Kac and Procesi. In particular we obtain a presentation of it by generators and relations, and a PBW-like theorem. Moreover, we give a direct proof that $\mathcal{F}_q[GL_n]$ is a Hopf subalgebra of $F_q[GL_n]$, and that $\mathcal{F}_q[GL_n]\Big|_{q=1} \cong U_{\mathbb{Z}}({\mathfrak{gl}_n}^*)$. We describe explicitly its specializations at roots of 1, say ε, and the associated quantum Frobenius (epi)morphism from $\mathcal{F}_\varepsilon[GL_n]$ to $\mathcal{F}_1[GL_n] \cong U_{\mathbb{Z}}({\mathfrak{gl}_n}^*)$. The same analysis is done for $\mathcal{F}_q[SL_n]$ and (as key step) for $\mathcal{F}_q[M_n]$.

This lecture is based on joint work with Fabio Gavarini, University Tor Vergata, Rome, Italy.

On normal and collectionwise normal locally convex topological vector spaces

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We consider the following question.

Let L be a topological locally convex vector space (LCS). Is it true that if L is normal, then L is collectionwise normal?

The answer to this question is unknown. The report contains some particular related results.

Theorem 1. *Let X be a convex subspace of an LCS L. Suppose that one of the following conditions holds:*

1) *$X \times X$ is a normal $\mathbb{R}$-factorizable space;*

2) *X is a normal $\mathbb{R}$-factorizable space and L is a Lindelöf Σ-space.*

Then X is collectionwise normal.

Corollary 2. *Let X be a linear subspace of an LCS L. Suppose that one of the following conditions holds:*

1) *$X \times X$ is a normal space and L is a product of Lindelöf Σ-spaces;*

2) *X is a normal space and L is a Lindelöf Σ-space.*

Then X is collectionwise normal.

On intersection of three embedded spheres in 3-space

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We study intersection of two polyhedral spheres without self-intersections in 3-space. We find necessary and sufficient conditions on sequences $x = x_1, x_2, \ldots, x_n$, $y = y_1, y_2, \ldots, y_n$ of positive integers, for existence of 2-dimensional polyhedra $f, g \subset \mathbb{R}^3$ homeomorphic to the sphere and such that

• $f - g$ has n connected components, of which can be numbered so that the i-th component has x_i neighbors in f and

• $g - f$ has n connected components, of which the i-th one has y_i neighbors in g.

Analogously we study intersection of *three* polyhedral spheres without self-intersections in 3-space.

See http://arxiv.org/abs/1012.0925

Some new classes of rigid polyhedra

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A polyhedron P in R^n is called *flexible* if it admits a continuous deformation keeping its hyperfaces as absolutely rigid (n-1)-dimensional "plates" with a variation only of some dihedral angles. If such a deformation doesn't exist the polyhedron is said *continuously rigid*. The question of checking of rigidity or flexibility of a given polyhedron is one of the main problems in the metric theory of polyhedra. In our talk we want to indicate two new classes of rigid polyhedra. The first class is composed by pyramids. A simplicial polyhedron is called *pyramid* if it has a vertex (named as the main one) joined with any other vertex by an edge. We accept this definition for polyhedra in the space of any dimension. At first we prove

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that in R^3 there exist pyramids of any topological type, orientable as well as nonorientable. For pyramids in $R^n, n > 3$, we admit that they can be pseudomanifolds too. Now we want to impose to pyramids some additional conditions, namely we suppose that

1) From any $(n-1)$-dimensional face of a considered pyramid P one can pass to any other $(n-1)$-dimensional face of P by a path intersecting only interior points of $(n-2)$-dimensional facets of P.

2) Let M_0 be a main vertex of a considered pyramid and $\langle M_{i_1}, \ldots, M_{i_{n-1}} \rangle$ be a $(n-2)$-dimensional facet of P where none of numbers $i_j, 1 \leq j \leq n-1$, is equal to 0. Then the $(n-1)$-dimensional tetrahedron with vertices $M_0, M_{i_1}, \ldots, M_{i_{n-1}}$ is not degenerate that is its $(n-1)$-dimensional volume is not equal to 0.

We will call pyramids with these properties as *pyramids of class A*. For them we have

Theorem 1 *Any pyramid P of class A in n-space is continuously rigid.*

The second class of rigid polyhedra is composed by bipyramids or suspensions. A *bipyramid or a suspension* is a polyhedron containing two vertices (called *poles*) which are not joined between them by an edge but both of them are joined to all other vertices by edges. These two special points are called poles of the suspension. This definition is valid for polyhedra in any n-space too. In 3-space there are orientable bypiramids of any topological genus (S. Lawrencenko) but one can show their existence in the nonorientable case too. We can prove the following

Theorem 2 *Let P be a suspension in R^3 of genus $g > 0$ and such that: 1) among the faces which are not incident to any pole there is at least one no degenerate face; 2) on the boundaries of the stars of poles there is no any connected part which could rotate around the axe passing through the poles. Then the suspention is continuously rigid.*

A generalization of this theorem to high dimensions is as follows:

Theorem 3 *Let P be an suspension in R^n such that it contains at least a hyperface not incident to any pole N or S and there is no any part of stars of the poles admitting a rotation around the axis NS. Then the suspension is continuously rigid.*

Infinite-dimensional spaces of probability measures

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In [1] there were investigated properties of infinite-dimensionality of spaces of type $\mathcal{F}(X)$, where $\mathcal{F} : Tych \to Tych$ is a covariant normal functor and X is a paracompact space. In particular, it was proved that a space $P_R(X)$ of all Radon probability measures on an infinite paracompact p-space X is strongly infinite-dimensional. Here we prove stronger versions of this theorem.

Given a Tychonoff space X let βX be its Cech-Stone compactification and

$$P_R(X) \subset P_\tau(X) \subset P_\sigma(X) \subset P(\beta X)$$

be the following subspaces of $P(\beta X)$:

$$\begin{aligned} P_R(X) &= \{\mu \in P(\beta X) : \\ &\quad \mu(K) = 1 \text{ for some } \sigma\text{-compact subset } K \subset X \subset \beta X\}; \\ P_\tau(X) &= \{\mu \in P(\beta X) : \\ &\quad \mu(K) = 0 \text{ for every compact subset } K \subset \beta X \setminus X\}; \\ P_\sigma(X) &= \{\mu \in P(\beta X) : \ \mu(K) = 0 \\ &\quad \text{for any closed } G_\sigma\text{-set } K \subset \beta X \text{ with } K \cap X = \emptyset\}. \end{aligned}$$

Theorem. *Let X be an infinite Tychonoff space. Then space $P_R(X)$, $P_\tau(X)$ and $P_\sigma(X)$ are strongly infinite-dimensional.*

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The Danto space and normal functors

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In the work [1], hereditary properties of some cardinal functions of spaces $O_\beta(X)$, where $O_\beta : Tych \to Tych$ - weakly normal functor acting in the category of Tychonoff spaces and their continuous mappings, are investigated. It is proved that the functor $O_\beta : Tych \to Tych$ doesn't preserve the spread, hereditary density, hereditary weakly density, hereditary π-weight, hereditary caliber, hereditary Shanin number, hereditary Lindelof number of Alexandroff arrow.

Let τ be an infinite cardinal number, X - a topological space and X' its subspace. The subspace X' is called τ monolithic [2] in X if for any $A \subset X'$ such that $|A| \leq \tau$ we have $[A]_X$ is compact with the weight τ. We say that X suppresses $X'[2]$ if from $\lambda \geq \tau$ and $A \subset X', |A| \leq 2^\lambda$ it follows that there exists $A' \subset X$ such hat $[A'] \supset A$ and $|A'| \leq \lambda$.

A topological space is called the Danto space [2] if for each infinite cardinal number τ there exists a dense subspace X' in X which is:

1) τ -monolithic in itself;

2) τ -suppressed by the space X simultaneously.

Theorem. Let X be a Danto space and $F : Comp \to Comp$ is a normal functor. Then $\varphi(F(X)) = \varphi(X)$, where $\varphi \in \{\chi, t, hd, h\pi w, hsh, hc, s\}$. Here χ - character, t -tightness,hd- hereditary density, $h\pi w$ - hereditary π -weight, hsh-hereditary Shanin number, hc- hereditary Souslin number, s-spread.

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Probability properties of minimal filling topologies for finite metric spaces

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The theory of minimal fillings for finite metric space was introduces by A.Ivanov and A.Tuzhilin as a generalisation of Shteiner minimal tree and Gromov minimal fillings problems. A filling G for a finite metric space (M, ρ) is a tree-graph (V, E) and a bijection B between elements of M and subset of V, such that $\forall m_1, m_2 \in M \rho(m_1, m_2) \leq d_G(B(m_1), B(m_2))$, where $d_G(p, q)$ is the length of the only existing path, connecting verti?es p and q. The problem to find a filling with a minimal possible sum of weights of all its edges is a difficult one, because for the moment there are only algorithms with the exponential complexity. In this work we introduce the probability measure on the space of topologies of minimal fillings for a special case of metric spaces and an algorithm for its computation. These gives a possibility to find a good approximation of the minimal filling, solving only a couple of linear-programming problems, corresponding to the most probable topologies.

On Topologies Generated by κ-Suslin Sets

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We show that topologies on λ^ω generated by κ-Suslin sets satisfy the Baire Category Theorem. Consequently, Projective Determinacy implies that so are topologies on ω^ω generated by effectively projective sets. Using this we establish some dichotomy theorems concerning σ-compactness of effectively projective sets.

Index of elliptic operators associated with diffeomorphisms of manifolds and uniformization

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Let M be a smooth manifold and $g : M \to M$ be a diffeomorphism. We develop elliptic theory for operators of the form

$$D = \sum_k D_k T^k : C^\infty(M) \longrightarrow C^\infty(M). \quad (1)$$

Here T is the shift operator $Tu(x) = u(g(x))$ along the orbits of g, D_k are pseudodifferential operators (ψDO) on M, and the sum is assumed to be finite. We obtain an index formula for operators of the form (1) in terms of topological invariants of the manifold and of the symbol of the operator using a new approach called *pseudodifferential uniformization*. The idea of this approach is to replace the operator (1), which is not local, by an elliptic pseudodifferential operator with the same index and then apply the celebrated Atiyah–Singer formula. We note here that the symbol of operator (1) is an element of the crossed product $C^\infty(S^*M) \rtimes \mathbb{Z}$ of the algebra of functions on the cosphere bundle by the action of the group $\mathbb{Z}$. Therefore, the final index formula is naturally formulated in terms of equivariant characteristic classes in cyclic cohomology of the crossed product.

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On continuous choice of continuous retractions onto nonconvex domains

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Typically, a creation of "generalized convexities", is usually related to an extraction of several principal properties of the classical convexity which are used in one of the key mathematical theorems or theories and, consequently deals with analysis and generalization of these properties in maximally possible general settings.

Principally another approach deals with a controlled omission of convexity in a set of basic theorems of multivalued analysis and topology. Roughly speaking, to each closed subset $P \subset B$ of a Banach space one can associates a numerical function, say $\alpha_P : (0, +\infty) \to [0, 2)$, the so-called function of nonconvexity of P. The identity $\alpha_P \equiv 0$ is equivalent to the convexity of P and the more α_P differs from zero the "less convex" is the set P.

Such classical results about multivalued mappings as the Michael selection theorem, the Cellina approximation theorem, the Kakutani-Glicksberg fixed point theorem, the von Neumann - Sion minimax theorem, etc. are valid with the replacement of the convexity assumption for values $F(x)$, $x \in X$ of a mapping F by some appropriate control of their functions of nonconvexity. Usually such a control means that α_P is "less" than 1. In this case the set P is said to be *paraconvex*.

In comparison with usual ideas of "generalized convexities", we never define in this approach, for example, a "generalized segment" joining $x \in P$ and $y \in P$. We look only for the distances between points z of the classical segment $[x, y]$ and the set and look for the ratio of these distances and the size of the segment. So the following natural question arises immediately: *Does paraconvexity of a set with respect to the classical convexity structure coincide with convexity under some generalized convexity structure?*

At this talk I provide an affirmative partial answer by using a suitable continuous choice of a retractions onto a paraconvex sets.

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Minimal linear Morse functions on the orbits in Lie algebras

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We consider the compact connected Lie group G and its linear representation in real linear space V. Then we consider some linear functional h on V and its restriction to some orbit of the representation of G. We shall deal with the following hypothesis: in the general case (which intend that h is the Morse function on the orbit) h is the minimal Morse function on the orbit; it means that for every k the quantity of critical points with index k equals to k-th Bettie number of the orbit. We shall prove this hypothesis in the following case: G is semisimple, representation is adjoint and the orbit is regular. In addition we shall describe the set of critical points and the kernels of Hessian of linear function on the orbit in the case of arbitrary representation.

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On the number of complement regions in submanifold arrangements

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Let us consider m–dimensional manifold M and the finite family $\{A_1, \dots, A_n\}$ of closed $(m-1)$–dimensional subsets. Let

$$f = |\pi_0(M \setminus \bigcup_{i=1}^{n} A_i)|$$

be the connected components number of the complement in M to the union of $A_1, \dots, A_n$. Let $F_n(M)$ be the set of numbers f for all possible arrangements of n subsets given type. The general question is to describe the sets $F_n(M)$ for arrangements of closed geodesics or totally geodesic surfaces in M. The sets $F_n(M)$ could be interesting in connection with Orlic and Solomon [1] statement that the region number in hyperplane arrangements equals to the cohomology ring dimension of the complement to complexified arrangement. N. Martinov [2] founded the sets of region numbers in real projective plane arrangements of lines and arrangements of pseudolines. Hence the sets of region numbers in standard sphere arrangements of big circles are also known.

Theorem. Let us consider arrangements of $(m-1)$–dimensional flat subtori in the m–dimensional flat torus T^m, arrangements of closed (non–simple) geodesics in the flat Klein bottle KL^2, arrangements of closed simple geodesics in the surface R of the tetrahedron, arrangements of hyperplanes in the hyperbolic metric of Lobachevsky space L^m and finally hyperplane arrangements with empty intersection of all hyperplanes in the real projective space P^m. Then

$$F_n(T^m) \supseteq \{n-m+1,\dots,n\} \cup \{l \in \mathbb{N} \mid l \geq 2(n-m)\} \qquad (1)$$
$$F_n(KL^2) = \{n+1\} \cup F_n(T^2) \quad \text{for } n \geq 2,\ F_1(KL^2) = \mathbb{N},$$
$$F_n(R) \subseteq \{n+1, 2n\} \cup \{l \in \mathbb{N} \mid l \geq 4n-6\} \quad \text{for } n \geq 3, \qquad (2)$$
$$F_n(L^m) = \left\{ f \in \mathbb{N} \;\middle|\; n+1 \leq f \leq \sum_{i=0}^{m} \binom{n}{i} \right\}$$

first four numbers of $F_n(P^m)$ for $n \geq 2m+5$ and $m \geq 3$ are:
$$(n-m+1)2^{m-1}, 3(n-m)2^{m-2}, (3n-3m+1)2^{m-2}, 7(n-m)2^{m-3}.$$

The inclusion (1) turns into equality at least for two–dimensional tori. The inclusion (2) turns into equality iff all tetrahedron faces are equal acute–angled triangles.

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When the set of links is finite?

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This talk is on a joint work Diarmuid Crowley and Steven C. Ferry. Let m and $p_1,\dots,p_r < m-2$ be positive integers. The set of links of codimension > 2, $E^m(\sqcup_{k=1}^r S^{p_k})$, is the set of smooth isotopy classes of smooth embeddings $\sqcup_{k=1}^r S^{p_k} \to S^m$. Haefliger showed that $E^m(\sqcup_{k=1}^r S^{p_k})$ is a finitely generated abelian group with respect to embedded connected summation and computed its rank in the case of knots, i.e. $r=1$. For $r>1$ and for restrictions on $p_1,\dots,p_r$ the rank of this group can be computed using results of Haefliger or Nezhinsky. Our main result determines the rank of the group $E^m(\sqcup_{k=1}^r S^{p_k})$ in general. In particular we determine precisely when $E^m(\sqcup_{k=1}^r S^{p_k})$ is finite. We also accomplish these tasks for framed links. Our proofs are based on the Haefliger exact sequence for groups of links and the theory of Lie algebras. The speaker was supported in part by President of the Russian Federation grant MK-3965.2012.1, by

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Interval identification systems of order 3 and plane sections of triply periodic surfaces

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The notion of interval identification systems is a generalization of interval exchange transformations and interval translation mappings. The same objects have also appeared in the theory of $\mathbb{R}$-trees as an instrument for describing the leaf space of a band complex. We study dynamical properties of such systems (including behavior of orbits) and applications of interval identification systems of order 3 to remaining open questions in Novikov's problem of asymptotic behavior of plane sections of triply periodic surfaces. Our main tools include the Rauzy induction and the Rips machine for band complexes.

Closed locally minimal networks on the surfaces of convex polyhedra

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A *network* is a geometric realization of an abstract graph, i.e. the vertices are represented by some points in a metric space and the edges are represented by curves connecting the corresponding points. If for each point of a network there is a neighborhood such that the network cannot be shortened by a deformation in this neighborhood then by definition the network is a locally minimal network. In the case of convex polyhedra this definition is equivalent to the following one.

Definition. A network on a convex polyhedron is called *closed locally minimal* if all the edges of the network are geodesics and at each node of the network precisely three edges meet at angles of $120°$.

What conditions are necessary and sufficient for a polyhedron to have a closed locally minimal network? There is no full answer. I present some related results. (This question is also open in the case of closed geodesics, but I do not touch this case.)

Theorem.(Ivanov, Tuzhilin, 1994) Suppose there exists a closed locally minimal network on polyhedron P. Then there exists a partition of the vertex set of P into several subsets such that in each subset the total Gaussian curvature of the vertices equals $\frac{k\pi}{3}$ for some $k = 1, \ldots, 5$, where k may be different for different subsets.

Theorem.(Strelkova, 2011) There exists a tetrahedron $ABCD$ with no closed locally minimal networks, though the curvatures are $K_A = \frac{5\pi}{3}$, $K_B = \frac{2\pi}{3}$, $K_C + K_D = \frac{5\pi}{3}$.

Theorem.(Strelkova, 2012) Fix any positive integers $k_1, \ldots, k_s$ such that $k_1 + \ldots + k_s = 12$, and denote by $\mathcal{M}$ the set of all polyhedra with exactly s vertices and curvatures $\frac{k_1\pi}{3}, \ldots, \frac{k_s\pi}{3}$ at the vertices. Then there exists an open dense subset $\mathcal{M}_0 \subset \mathcal{M}$ such that on each polyhedron from $\mathcal{M}_0$ there exists a closed locally minimal network.

Local **Index Theorem.**

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It is well known that the cyclic (co)-homology of Banach algebras associated to topological spaces do not reflect well the spaces themselves. Different solutions were proposed to obtain meaningful results in this case. One solution is due to A. Connes, who introduced entire cyclic co-homology for Banach algebras. His definition of entire co-homology is based on a growth condition of the norm of the infinite components of the co-chains over the algebra. This definition fits perfectly with the Novikov conjecture problem on manifolds with hyperbolic fundamental group. A different solution is due to M. Puschnigg, who introduced the notion of local cyclic homology of Frechet algebras; his construction is based on the system of pre-compact subsets of the algebra. In this lecture we introduce a new notion of local cyclic homology based on the supports of the chains. We show that this notion adapts naturally to the case of smoothing operators on smooth, Lipschitz or quasi-conformal manifolds and allows one to reinterpret the Connes-Moscovici local index theorem. Our notion of local cyclic homology inserts into the Hochschild and cyclic complex (over the algebra of smoothing operators) the ideas of the Alexander-Spanier homology construction. For any pseudo-differential elliptic operator on a smooth manifold, our definition of local cyclic homology allows one to relate in a more transparent way the Connes-Karoubi-Chern character, associated to the operator, with the Alexander-Spanier homology of the space. To do this, we construct a homological Connes-Karouby-Chern type character based on the residue smoothing operator R = P - e, where P and e are idempotents; the operator R was defined by Connes-Moscovici in their original paper at p. 353. Our Connes-Karoubi-Chern type character lives in the cyclic complex of the algebra of smoothing operators localized to the separable ring L = C + Ce. In the second part of the lecture we compute the local Hochschild homology of the algebra of Hilbert-Schmidt operators on locally compact countable homogeneous simplicial complexes and we explicit the parallelism between our definition of local Hochschild homology with the Alexander-Spanier homology.

Hirzebruch genera on homogeneous spaces

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We aim to present how the notion of universal toric genus can be applied in the case of homogeneous spaces to obtain the important results on some famous Hirzebruch genera such as Krichever genus, signature, elliptic genus and any genus defined by an odd power series.

Universal toric genus can be defined for any stable complex manifold with an equivariant torus action. It as an element of the complex cobordism ring of the classifying space of the acting torus. When the torus action has only isolated fixed points the toric genus can be localized meaning that it can be expressed in terms of signs and weights at the fixed points for the given torus action. When composed with a Hirzebruch genus, the toric genus gives arise to an equivariant genus which carries important information, such as rigidity or even triviality, on the initial Hirzebruch genus.

We explain how this approach can be used in the case of compact homogeneous spaces of positive Euler characteristic equipped with the canonical action of the maximal torus and an invariant almost complex structure. We prove that Krichever genus is rigid for this action just using representation theory for Lie groups. We also prove using representation theory that any Hirzebruch genus given by an odd power series is trivial on a large class of homogeneous spaces what then holds for the signature, $\hat{A}$-genus and the elliptic genus as well.

The talk is based on the joint work with Victor M. Buchstaber (Toric genera of homogeneous spaces and their fibrations, International Mathematics Research Notices, 2012).

Torsion in gauge groups

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Let M be a simply-connected 4-manifold, let $P \to M$ be a principal $SU(2)$-bundle, and let $G(P)$ be the gauge group of this principal bundle. For p an odd prime we calculate the mod-p homology of the classifying space of $G(P)$ in many cases, including when $M = S^4$. These calculations are of interest to mathematical physics and Donaldson theory.

Computing dimensions by the experimental data

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Statement of the problem. Let Ω be a compact metric space with a metric ρ. Let μ be a Borel probability measure on Ω. Let $\boldsymbol{\xi_0}, \boldsymbol{\xi_1}, \ldots, \boldsymbol{\xi_n}$ be independent random variables taking values in Ω and identically distributed with a common law μ.

We want to evaluate the reciprocal of the dimension of μ.

The aim of this research is to construct an estimator to achieve efficiency $O(n^{-c})$ for some measures, where $c > 0$ is a constant.

The estimator is defined as follows:

$$\eta_n^{(k)} = k\left(r_n^{(k)} - r_n^{(k+1)}\right), \tag{1}$$

where

$$r_n^{(k)} = r_n^{(k)}(\rho) = -\frac{1}{n+1}\sum_{j=0}^{n} \ln\left(\min_{i:i\neq j}{}^{(k)} \rho^{(m)}(\boldsymbol{\xi_i}, \boldsymbol{\xi_j})\right), \qquad (2)$$

and we have just introduced the following notation: for any ordered set $x_1 \leq x_2 \leq \cdots \leq x_N$, $\min^{(k)}\{x_1, \ldots, x_N\}$ is defined to be x_k.

Euclidean spaces. Let Ω be an s-dimensional compact manifold in $\mathbb{R}^d$. Let μ be a Lebesgue probability measure on Ω. In [1] was shown that the variance and the bias of the estimator (1) has the order $O(n^{-c})$, if the measure of any ball is the smooth function, where $c > 0$ is a constant.

Sequence spaces. Let $\Omega = \mathcal{A}^{\mathbb{N}}$ be a space of all right-sided infinite sequences drawn from a finite alphabet $\mathring{A}$, where $\mathbb{N} = \{1, 2, \dots\}$. Let μ be a shift-invariant probability measure on Ω and $\rho(\boldsymbol{x}, \boldsymbol{y})$ be a metric on Ω.

If ρ is bi-Lipschitz equivalent to the metric (3), where $\lambda(t) = 0$, then the estimator (1) is the estimator of the reciprocal of the entropy.

In [1] was also shown that the variance of the estimator (1) has the order $O(n^{-c})$ for certain classes of measures and metrics, where $c > 0$ is a constant.

However, the problem of finding the bias is much more difficult. In [2] considered the estimator (1) for the metric (3), where $\lambda(t) = 0$, and Markov measures and proved that the estimator (1) asymptotically unbiased only if the logarithms of some transition probabilities are rationally incommensurable. The bias was a periodic function with a period proportional to $\log n$ if the logarithms of transition probabilities are rationally commensurable.

Thus, the efficiency of the entropy estimator is determined by metric's properties. We introduce a new metric on Ω

$$\rho(a\boldsymbol{x}, b\boldsymbol{y}) = \begin{cases} e^{-1}\rho_i(\boldsymbol{x}, \boldsymbol{y}), & a = b; \\ e^{-\lambda(-\ln \rho(\boldsymbol{x}, \boldsymbol{y}))}, & a \neq b; \end{cases} \qquad (3)$$

where $\lambda(t) = \min\{1, \beta t\}$ and $0 < \beta < 1$; and prove that for a Bernoulli measure μ, there exist β such that

$$\lim_{n\to\infty} \mathsf{E}\eta_n^{(k)} = 1/h,$$

where h is the entropy of μ and k is a constant.

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Burau and Jones representations of braid groups — the relation

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The famous Burau representation of braid groups is given in its classic version as a matrix representation. The Burau representation is known to have the same kernel as that of a certain weak version of the Jones representation (into the Temperley-Lieb algebra). In fact the two representations are practically the same mathematical object — if properly understood. We will discuss how the Temperley-Lieb version may be used to study the original Burau reperesentation.

Reidemeister numbers for residually finite groups

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We prove for residually finite groups the following long standing conjecture: the number of twisted conjugacy classes ($g \sim hg\phi(h^{-1})$) of an automorphism ϕ of a finitely generated group is equal (if it is finite) to the number of finite dimensional irreducible unitary representations being invariant for the dual of ϕ.

Also, we prove that any finitely generated residually finite non-amenable group has the R_∞ property (any automorphism has infinitely many twisted conjugacy classes). This gives a lot of new examples and covers many known classes of such groups.

The research was partially supported by RFBR Grants 10-01-00257-a and 11-01-90413-Ykp-ф-a and Grant 2010-220-01-077 (contract 11.G34.31.0005) of the Russian Government.

The talk is based on a joint paper with A.Fel'shtyn (arXiv: 1204.3175).

Complex geometry of moment-angle-complexes

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In this talk we associate compact manifold Z_K to every simplicial complex K, and construct complex structure on it. This manifolds generalise well-known Hopf and Calabi-Eckman manifolds. We discuss complex geometry of these manifolds, in particular we describe their Dolbeault cohomology, field of meromorphic functions and complex submanifolds.

Countable Dense Homogeneity

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We prove that a connected, countable dense homogeneous space is n-homogeneous for every n, and strongly 2-homogeneous provided it is locally connected. We also present an example of a connected and countable dense homogeneous space which is not strongly 2-homogeneous. We prove that a countable dense homogeneous space has size at most continuum. If it moreover is compact, then it is first-countable under the Continuum Hypothesis. We also construct under the Continuum Hypothesis an example of a hereditarily separable, hereditarily Lindelöf, countable dense homogeneous compact space of uncountable weight. We also discuss locally compact separable metrizable spaces with a finite number of types of countable dense sets and prove a structure theorem for them. Some of the presented results were obtained with A.V. Arhangelskii and M. Hrusak.

Generalized Cantor manifolds

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The connection between generalizations of Cantor manifolds and homogeneous continua will be discussed. Some result concerning the question whether any homogeneous metric continuum is a V(n)-continuum in the sense of Alexandroff will be also presented.

Cyclic generalizations of hyperbolic 3-manifolds constructed from regular polyhedra

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Various examples of three-dimensional spherical, Euclidean, or hyperbolic manifolds arise from pairwise isometrical identifications of faces of convex regular polyhedra in the corresponding 3-spaces: $\mathbb{S}^3$, $\mathbb{E}^3$, or $\mathbb{H}^3$.

There are eight manifolds, arising from a regular hyperbolic dodecahedron with dihedral angled $2\pi/5$. One of them is the Weber – Seifert manifold (1933) that is the 3-fold cyclic branched covering of S^3 branched over the Whitehead link. Also, there are six manifolds, arising from a regular hyperbolic icosahedron with dihedral angles $2\pi/3$. One of them is the Fibonacci manifold, uniformized by the Fibonacci group $F(2,10)$, is the 5-fold cyclic branched coverings of S^3, branched over the figure-eight knot. Cyclic generalizations of above manifolds were investigated by many authors.

Two other manifolds, arising from a regular $2\pi/3$-icosahedra, can be presented as 3-fold cyclic branched coverings of the lens space $L(3,1)$. Cyclic generalizations of these manifolds are constructed and studded in [1,2].

The method from [1] gives a way for a polyhedral construction for n-fold cyclic branched coverings of lens spaces $L(p,q)$. Conditions for existence of cyclic branched coverings of 3-manifolds were obtained in [3].

In the present talk we will discuss results and methods from [1] and [2].

[6]Supported by RFBR (grant 10-01-00642) and by the grant for Russian Leading Scientific Schools (NSh.-5682.2012.1).

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Geometrical realization of γ-vectors of 2-truncated cubes.

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The talk is devoted to the family of combinatorial polytopes that can be obtained from a cube by sequence of truncations of codimension 2 faces (below called truncated cubes). Every 2-truncated cube P is a flag simple polytope and it was shown that there exists a flag simplicial complex Δ_P such that $f(\Delta_P) = \gamma(P)$. Therefore, γ-vectors of 2-truncated cubes satisfy Frankl-Furedi-Kalai inequalities. The class of 2-truncated cubes include many well-known classes of simple polytopes (flag nestohedra, graph-associahedra and graph-cubeahedra). It was shown that γ-, g-, h-, f-vectors of associahedra, cyclohedra, permutohedra and stellohedra are the sharp bounds for γ-, g-, h-, f-vectors of certain subclasses of graph-associahedra.

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Cohomology of Lie algebras of vector fields on orbifold S^1/Z_2

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Let S^1 be the unit circle in the complex plane. I.M. Gelfand and D.B. Fuchs proved that the cohomology algebra of the Lie algebra of vector fields on S^1 is isomorphic to the tensor product of the polynomial ring with one generator of degree two and the exterior algebra with one generator of degree three. We compute the cohomology of the Lie algebra of vector fields on the one-dimensional orbifold S^1/Z_2, where Z_2 acts on S^1 by reflection in the Ox axis and S^1/Z_2 is the orbit space. We prove that the cohomology algebra of the Lie algebra of vector fields on S^1/Z_2 is isomorphic to the tensor product of the exterior algebra with two generators of degree one and the polynomial algebra with one generator of degree two.

The considered Lie algebra is a subalgebra of the Lie algebra of vector fields on S^1. It should be noted that the Gelfand-Fuchs generators vanish under the restriction to the subalgebra.

Minimal sets of conformal foliations

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New our results will be presented on the investigation of the structure of conformal foliations.

Let $(M, \mathcal{F})$ be an arbitrary smooth foliation. Remind that a subset of a manifold M is called a saturated whenever it is the union of some leaves of a foliation $(M, \mathcal{F})$. A nonempty saturated subset $\mathfrak{M}$ of M is called a minimal set of $(M, \mathcal{F})$, if each leaf from $\mathfrak{M}$ is dense in $\mathfrak{M}$. The leaf L of a foliation $(M, \mathcal{F})$ is called closed if L is a closed subset of M. Any closed leaf of $(M, \mathcal{F})$ is a minimal set.

By definition, an attractor of a foliation $(M, \mathcal{F})$ is nonempty saturated subset $\mathcal{M}$, if there exists a saturated open neighborhood $Attr(\mathcal{M})$ such that the closure of every leaf from $Attr(\mathcal{M})$ includes $\mathcal{M}$. If an addition $Attr(\mathcal{M}) = M$, then the attractor $\mathcal{M}$ is called global.

We have proved that every codimension $q \geq 3$ conformal foliation $(M, \mathcal{F})$ either is Riemannian or has a minimal set that is an attractor of $(M, \mathcal{F})$, and the restriction of the foliation to the attraction basin is a $(Conf(S^q), S^q)$-foliation. We do not assume that M is compact.

We have shown that each proper non-Riemannian codimension $q \geq 3$ conformal foliation has a closed leaf that is an attractor.

Sufficient conditions for the existence of a global attractor of a conformal foliation has been found. The structure of the global attractors and foliations $(M, \mathcal{F})$ have been investigated.

We have proved also that every conformal foliation $(M, \mathcal{F})$ on a compact manifold M either a Riemannian foliation or a $(Conf(S^q), S^q)$-foliation with a finite family of attractors. They are all minimal sets of this foliation.

Examples of conformal foliations with exceptional and exotic global attractions are constructed.

This work was supported by the Russian Foundation for Basic Research, grant N 10-01-00457.

Some properties of quotient functors

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In the paper it is shown that if we consider the functor F under the natural restrictions then property of being approximated from within by Tominaga class K of metric compact is preserved for $F(X)$ and class $F(K)$. Here the functor F acts in the category *Comp*.

Definition. It is said that the compact X is strongly approximated from within by Tominaga class K if there exists a sequence $C_1 \subset C_2 \subset ... \subset X$, $C_i \in K$ such that $X = \bigcup C_i$ and satisfied conditions: for any $\varepsilon > 0$ there exists C_i and the mapping $f_\varepsilon : X \to X$ such that for any $s > j$ the restriction $f_\varepsilon|_{C_s} : C_s \to C_s$ is continuous and $\rho(x, f_\varepsilon(x)) < \varepsilon$ for any $x \in C_s$, where ρ is a metric of X; for any $x \in \bigcap(\overline{X})$ there exists neighborhood U_x such that its component of connectedness containing x is contained in some C_i

Example. The Warsaw circle is an example of the space satisfying the definition, where class K consists of segments.

Theorem [1]. If the compact X is approximated on the class of Peano continua with a fixed point property then the compact X has fixed point property.

Theorem. Let F be the space of quotient functors of finite degree n[2]. If compact X is strongly approximated from within by Tominaga class K of Peano continua then the compact $F(X)$ is strongly approximated from within by Tominaga class $F(K)$.

Corollary. If the condition of theorem is satisfied and $F(K)$ consists of Peano continua with the fixed point property then $F(X)$ possesses the fixed point property

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Geometry of Dold Manifolds

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Dold manifolds are smooth manifolds that were introduced by Dold to obtain a family of generators for the unoriented cobordism ring. Specifically, $P(m, n)$ is the $m + 2n$ dimensional smooth manifold that is obtained as the identification space of $S^m \times \mathbb{C}P^n$ under the action of the fixed point free involution $\tau(x, [z_0, z_1, \ldots, z_n]) = (-x, [\overline{z_0}, \overline{z_1}, \ldots, \overline{z_n}])$. For example, $P(m, 0)$ is the familiar $\mathbb{R}P^m$. In this talk we will discuss new contributions to the geometry of these manifolds, specifically to the questions of their span and parallelizability, that are due mainly to J. Korbaš and his student P. Novotný. Similar questions for other families of manifolds that generalize the real projective spaces will be briefly discussed.

About correct solvability for the nonlinear equations

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The concept of the correct (Hadamard) solvability for linear equations is well known (see, for example, [1]). In the present report the analogue of this concept is constructed for the nonlinear equations. As an example it is considered the initial-boundary problem for the quasilinear parabolic equation.

Definition: Let X, Y be topological spaces, $f\colon X \to Y$ - mapping and $y \in Y$ - be the fixed element.The nonlinear equation

$$f(x) = y \tag{1}$$

will be correctly solvable, if for any open neighbourhood U of the set of solutions of the equation (1) in X there is an open neighbourhood V of the point y in Y such that for any $y' \in V$ the set of solutions of the equation $f(x) = y'$ contains in U.

The establishment of the fact that the equation is correctly solvable is important for research the problems described by this equation.

Theorem: If X, Y - metric spaces, then for any continuous proper mapping $f\colon X \to Y$ the equation $f(x) = y$ is correctly solvable.

Let's consider a initial-boundary problem. Let Ω be a bounded domain of $\mathbb{R}^n$ with smooth border $\partial\Omega$. Let $T > 0$ be arbitrary number, $Q_T = (0,T) \times \Omega$.

$$\frac{\partial v}{\partial t} - \sum_{i,j=1}^{n} a_{ij}(t,x,v,\frac{\partial v}{\partial x_1},...,\frac{\partial v}{\partial x_n})\frac{\partial^2 v}{\partial x_i \partial x_j} + g(t,x,v,\frac{\partial v}{\partial x_1},...,\frac{\partial v}{\partial x_n}) = h(t,x), \tag{2}$$

where $(t,x) \in Q_T$ and $a_{ij}(t,x,v,\frac{\partial v}{\partial x_1},...,\frac{\partial v}{\partial x_n})$ - continuous on all variables,

$$v(t,x) = 0, \quad (0 < t \leqslant T, x \in \partial\Omega), \tag{3}$$

$$v(0,x) = v_0(x), \quad x \in \bar{\Omega}, \quad v_0 \mid_{\partial\Omega} = 0. \tag{4}$$

Theorem: The operator equation corresponding to the initial-boundary problem (2) - (4) for the quasilinear parabolic equation in the presence of a priori estimate is correctly solvable.

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Об общем виде равномерно непрерывного функционала на C_p-пространстве

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Тематика представляемой работы относится к так называемой C_p-теории, объектом изучения которой является пространство $C_p(X)$ всех непрерывных вещественнозначных функций на пространстве X, наделённое топологией поточечной сходимости. Здесь и далее X – тихоновское пространство. Известно, что для каждого ненулевого непрерывного линейного функционала l на пространстве $C_p(X)$ найдутся конечные множества $\{x_1, \ldots, x_n\} \subset X$ и $\{\lambda_1, \ldots, \lambda_n\} \subset \mathbb{R}$ такие, что $l(f) = \lambda_1 f(x_1) + \ldots + \lambda_n f(x_n)$ для всех $f \in C_p(X)$. Другими словами, найдётся линейная функция $L : \mathbb{R}^n \to \mathbb{R}$ такая, что $l(f) = L\left(f(x_1), \ldots, f(x_n)\right)$. Эта формула демонстрирует общий вид непрерывного линейного функционала на пространстве $C_p(X)$. Поскольку непрерывный линейный функционал является частным случаем равномерно непрерывного функционала, то естественно возникает вопрос, каким будет общий вид последнего. Этот вопрос интересен ещё и тем, что его решение может быть полезным для получения результатов, описывающих свойства u-эквивалентных пространств.

Прежде чем сформулировать главный результат работы, дадим ещё одно определение. Через $U_p(X)$ обозначим пространство всех равномерно непрерывных функционалов на пространстве $C_p(X)$ с топологией поточечной сходимости.

Теорема. Для каждого не равного константе функционала $\varphi \in U_p(X)$ существуют последовательность $\{x_k\} \in X^{\mathbb{N}}$ (или конечное множество $\{x_1, \ldots, x_n\} \subset X$) и равномерно непрерывная функция $\Phi : \mathbb{R}^{\mathbb{N}} \to \mathbb{R}$ (соответственно $\Phi : \mathbb{R}^n \to \mathbb{R}$) такие, что

а) $\varphi(f) = \Phi\left(\{f(x_k)\}\right)$ (соответственно $\varphi(f) = \Phi(f(x_1), \ldots, f(x_n))$) для всех $f \in C_p(X)$;

б) Многозначное отображение пространства $U_p(X)$ в X, ставящее в соответствие каждому $\varphi \in U_p(X)$ множество членов последовательности $\{x_k\}$ (соответственно множество $\{x_1, \ldots, x_n\}$), полунепрерывно снизу.

Верхний центральный ряд группы подстановок рядов над $\mathbb{Z}_2$

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В 1954 году Jennings ввёл группу $J(k)$ всевозможных формальных степенных рядов переменной x с коэффициентами в кольце k, имеющих вид $f(x) = x(1 + \alpha_1 x + \alpha_2 x^2 + ...)$, с операцией подстановки ряда в ряд. Обсуждается случай кольца $k = \mathbb{Z}_2$. Leedham-Green и McKay вычислили нижний центральный ряд этой группы. В работе вычисляется верхний центральный ряд.

Введём серию подмножеств $J(\mathbb{Z}_2)$, являющимися в нашем случае подгруппами,

$$J(a,b,c) = \big\{f(x) = x\big(1 + \sum_{i=0}^{\infty} \alpha_{a+2i} x^{a+2i} + \sum_{i=0}^{\infty} \alpha_{b+4i} x^{b+4i} + \sum_{i=0}^{\infty} \alpha_{c+4i} x^{c+4i}\big)\big\},$$

$$J_=(a,b,c) = \big\{f(x) \in J(a,b,c) \ : \ \alpha_a = \alpha_b = \alpha_c\big\},$$

$$J_0(a,b,c) = \big\{f(x) \in J(a,b,c) \ : \ \alpha_a + \alpha_b + \alpha_c = 0\big\},$$

где a – нечётное число, b – делится на 4, c – при делении на 4 даёт остаток 2.

Leedham-Green и McKay показали, что коммутантом группы $J(\mathbb{Z}_2)$ является подгруппа $J_=(3,4,6)$. Остальные члены верхнего центрального ряда последовательно описывают

Теорема 1. *При $c = 2a$, $4 \le a+1 \le b \le c-2$ коммутантом группы $J_=(a,b,c)$ является группа $J_0(a+b+2, c+2a+4, c+2b+4)$.*

Теорема 2. *При $c = 2a$, $4 \le a+1 \le b \le c-2$ коммутантом группы $J_0(a,b,c)$ является группа $J_=(b+a, c+2a, c+2b)$.*

Явно описываются факторы верхнего центрального ряда

Теорема 3. *Группа $J(\mathbb{Z}_2)$ является группой с тремя образующими и отображение*

$$\varphi\colon J(\mathbb{Z}_2) \to \mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_4 :$$
$$\varphi(f) = (\alpha_1, \alpha_4 + \alpha_6 + \alpha_2\alpha_4, \alpha_1^2 + \alpha_2^2 + 2\alpha_3 + 2\alpha_4).$$

является гомоморфизмом абелинизации.

Теорема 4. *Группа $J_=(3,4,6)$ является группой с шестью образующими и отображение*

$$\varphi\colon\ J_=(3,4,6) \to (\mathbb{Z}_2)^5 \oplus \mathbb{Z}_8\ :$$
$$\varphi(f) = \big(\alpha_5, \alpha_7+\alpha_8, \alpha_{10}, \alpha_{12}+\alpha_3\alpha_8, \alpha_{14}+\alpha_{12}+\alpha_7+\alpha_3\alpha_{10},$$
$$4(\alpha_9+\alpha_{16}+\alpha_{18}+\alpha_8+\alpha_3\alpha_{14}+\alpha_3\alpha_8+\alpha_8\alpha_{10})+2\alpha_8^2+\alpha_3^4\big).$$

является гомоморфизмом абелинизации.

О функторах со свойством Катетова

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Мы рассматриваем только ковариантные функторы в категории компактов. Функтор $\mathcal{F}$ обладает свойством Катетова (K-свойством), если для любого компакта X наследственная нормальность $\mathcal{F}(X)$ влечет метризуемость X. По теореме Катетова о кубе компакта функтор возведения в куб обладает K-свойством. Начало исследованиям по распространению теоремы Катетова на различные классы функторов положил В.В.Федорчук [1], доказавший, что K-свойством обладают все нормальные функторы степени ≥ 3. Дальнейшие результаты в этом направлении были получены Т.Ф.Жураевым, А.П.Комбаровым, автором, Е.В.Кашубой и др. Во всех упомянутых исследованиях на функтор $\mathcal{F}$ налагаются требования нормальности или полунормальности и, по сути, речь идет о функторах конечной степени.

Напомним, что функтор $\mathcal{F}$ называется полунормальным, если он непрерывен, сохраняет мономорфизмы, пересечения, точку и пустое множество. Назовем функтор $\mathcal{F}$ финитно мономорфным, если он всякий мономорфизм конечного компакта переводит в мономорфизм. Заметим, что любой функтор сохраняет мономорфность отображения конечного компакта в нульмерный компакт. В то же время, существует не финитно мономорфный непрерывный функтор степени 2, сохраняющий точку, пересечения и пустое множество.

Будем говорить, что финитно мономорный функтор $\mathcal{F}$ сохраняет пустое пересечение, если для любых двух конечных непересекающихся подмножеств $A, B \subset X$ $\mathcal{F}(A) \cap \mathcal{F}(B) = \emptyset$. Оказывается, что для финитно мономорфных функторов конечной степени, сохраняющих точку и пустое пересечение, справедливы все полученные к настоящему времени утверждения о K-свойстве. В частности, верна следующая теорема из [2]: любой такой функтор степени больше 3 обладает K-свойством. В связи с полученными результатами возникает ряд вопросов. Например:

Верно ли, что всякий финитно мономорфный функтор бесконечной степени мономорфен?

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Целочисленные решетки переменных действия для систем типа "Сферический маятник"

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Пусть (M^{2n}, ω, H) — интегрируемая гамильтонова система с n степенями свободы, а $F_1, \cdots, F_n$ — ее первые интегралы, где $F_1 = H$. Пусть $\Phi = (F_1, \cdots, F_n) : M^{2n} \to \mathbb{R}^n$ — *отображение момента.*

Пусть существует и фиксирована 1–форма α на связной открытой области P^{2n} в M^{2n}, такая что $d\alpha = \omega$. Пусть все неособые слои Лиувилля, попавшие в область P^{2n}, компактны и связны. Тогда множество точек в $\Phi(P^{2n})\backslash\Sigma \subset \mathbb{R}^n$, образованное пересечениями n поверхностей уровня $\{\xi \in \Phi(P^{2n})\backslash\Sigma|\ I_1(\xi) = c_i,\ i = 1, \cdots, n,\ c_i \in \mathbb{Z}\}$ функций I_i, определенных по формулам $I_i(\xi) = \frac{1}{2\pi}\int\limits_{\gamma_i} \alpha$, где $\gamma_1(\xi), \ldots, \gamma_n(\xi)$ – 1-циклы на торе Лиувилля T_ξ, образующие базис группы гомологий $H_1(T_\xi)$ и непрерывно зависящие от ξ, α — какая-либо 1-форма в $U(\xi)$, такая, что $d\alpha = \omega$, назовем *целочисленной решеткой $\mathcal{R}$ переменных действия* (или просто *решеткой*).

Система "сферический маятник" хорошо известна. В частности, для нее аналитически описаны: функция Лагранжа, интегралы движения, эффективный потенциал и т.д. Однако исследование топологических инвариантов этой интегрируемой системы встречается гораздо реже, при этом анализ проводится, как правило, в связи с задачами квантовой физики. В данной работе полностью исследована топология системы "сферический маятник" выведены формулы для переменных действия, описаны кривые и точки, образующие бифуркационную диаграмму, описан образ отображения момента, исследован тип особых точек ранга 0, а также поведение линий уровня переменных действия на образе отображения момента и построена решетка переменных действия. Приведен алгоритм вычисления по решетке меток и матрицы монодромии изолированного особого значения.

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Примарные разложения узлов в утолщенных поверхностях

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Пусть $K \subset F \times I$ — узел в утолщенной поверхности, то есть простая замкнутая кривая K в прямом произведении замкнутой ориентируемой поверхности F на отрезок I. На множестве всех таких узлов введём три типа редукций.

Редукция типа 1. Пусть $A \subset F \times I$ — послойное разбивающее кольцо, трансверсально пересекающее узел $K \subset F \times I$ в двух точках. Тогда редукция типа 1 состоит в разрезании многообразия $F \times I$ по кольцу A и заклеивании двух получившихся копий этого кольца двумя ручками индекса 2 с тривиальными дугами в них. В результате получаются два узла $K_1 \subset F_1 \times I$ и $K_2 \subset F_2 \times I$.

Редукция типа 2 (дестабилизация) состоит в разрезании многообразия $F \times I$ по послойному кольцу, не пересекающему узла K, и заклеивании двух копий этого кольца ручками индекса 2.

Редукция типа 3. Пусть A_1, A_2 — такая пара непересекающихся послойных колец в $F \times I$, что их объединение разбивает $F \times I$, и узел K трансверсально пересекает каждое кольцо в одной точке. Тогда редукция типа 3 состоит в разрезании многообразия $F \times I$ по кольцам A_1, A_2 на две части и склеивании копий этих колец на краях частей по обращающим ориентации гомеоморфизмам так, чтобы получились два узла $K_1 \subset F_1 \times I$ и $K_2 \subset F_2 \times I$.

В работе [5] было показано, что если узел $K \subset F \times I$ является гомологически тривиальным, то его разложение в кольцевую связную сумму примарных узлов существует и единственно. В общем случае единственности нет, то есть применение редукций типа 1 к гомологически нетривиальному узлу может дать различные наборы примарных слагаемых. Единственности можно добиться за счет дальнейшего разложения слагаемых с помощью редукций типов 2 и 3. Следующая теорема получена с использованием методов теории корней, разработанной в [2].

Теорема 1. *Процесс последовательного применения нетривиальных редукций типов 1 – 3 к произвольному узлу $K \subset F \times I$ конечен. Получающийся в результате набор узлов в утолщенных поверхностях зависит только от исходного узла с точностью до удаления тривиальных узлов в утолщенных сферах.*

Разрешение операций дестабилизации позволяет перейти от множества всех узлов в утолщенных поверхностях к его фактор-множеству, которое совпадает со множеством всех виртуальных узлов [1, 3, 4]. При этом редукции типов 1 и 3 индуцируют операцию связного суммирования виртуальных узлов. Следующий результат является прямым следствием теоремы 1.

Теорема 2. *Любой виртуальный узел раскладывается в связную сумму нескольких примарных и тривиальных виртуальных узлов. При этом примарные слагаемые такого разложения определены однозначно, то есть зависят только от исходного виртуального узла.*

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Гомотопические свойства дифференциальных модулей с F_∞-симплициальными гранями

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Прежде всего напомним, что дифференциальным модулем с симплициальными гранями (X, d, ∂_i) называют дифференциальный биградуированный модуль (X, d), где $X = \{X_{n,m}\}$, $n, m \in \mathbb{Z}$, $n \geqslant 0$, $d : X_{*,\bullet} \to X_{*,\bullet-1}$, рассматриваемый вместе с семейством отображений дифференциальных модулей $\partial_i : X_{n,\bullet} \to X_{n-1,\bullet}$, $0 \leqslant i \leqslant n$, $n \geqslant 0$, называемых симплициальными гранями, для которых выполнены симплициальные коммутационные соотношения

$$\partial_i \partial_j = \partial_{j-1} \partial_i : X_{n,\bullet} \to X_{n-2,\bullet}, \quad i < j. \tag{1}$$

Введем теперь понятие дифференциального модуля с F_∞-симплициальными гранями, которое является с точностью до высших гомотопий аналогом указанного выше понятия дифференциального модуля с симплициальными гранями.

Пусть задано произвольное целое число $n \geqslant 0$. Любой набор целых неотрицательных чисел $(i_1, \ldots, i_k)$, где $0 \leqslant i_1 < \cdots < i_k \leqslant n$, будем называть упорядоченным набором. Пусть Σ_k – симметрическая группа перестановок множества из k элементов, и пусть $(i_1, \ldots, i_k)$ – произвольный упорядоченный набор. Для любой перестановки $\sigma \in \Sigma_k$ рассмотрим набор $(\sigma(i_1), \ldots, \sigma(i_k))$, где σ действует на набор $(i_1, \ldots, i_k)$ обычным образом, т.е. путем перестановки чисел из этого набора местами. Для указанного набора $(\sigma(i_1), \ldots, \sigma(i_k))$ определим набор $(\widehat{\sigma(i_1)}, \ldots, \widehat{\sigma(i_k)})$, где $\widehat{\sigma(i_s)} = \sigma(i_s) - \alpha(\sigma(i_s))$, $1 \leqslant s \leqslant k$, и $\alpha(\sigma(i_s))$ – количество чисел из набора $(\sigma(i_1), \ldots, \sigma(i_s), \ldots \sigma(i_k))$, стоящих справа от числа $\sigma(i_s)$ и меньших его.

Определение. Дифференциальным модулем с F_∞-симплициальными гранями $(X, d, \widetilde{\partial})$ будем называть дифференциальный биградуированный модуль (X, d), где $X = \{X_{n,m}\}$, $n, m \in \mathbb{Z}$, $n \geqslant 0$, $d : X_{*,\bullet} \to X_{*,\bullet-1}$, рассматриваемый вместе с семейством отображений модулей $\widetilde{\partial} = \{\partial_{(i_1,\ldots,i_k)} : X_{n,\bullet} \to X_{n-k,\bullet+k-1}\}$, $n \geqslant 0$, $(i_1, \ldots, i_k)$ – любой упорядоченный набор, называемых F_∞-симплициальными гранями, для которых выполнены соотношения

$$d(\partial_{(i_1,\ldots,i_k)}) = \sum_{\sigma \in \Sigma_k} \sum_{I_\sigma} (-1)^{\mathrm{sign}(\sigma)+1} \partial_{(\widehat{\sigma(i_1)},\ldots,\widehat{\sigma(i_m)})} \partial_{(\widehat{\sigma(i_{m+1})},\ldots,\widehat{\sigma(i_k)})}, \tag{2}$$

где I_σ – множество всех разбиений набора чисел $(\widehat{\sigma(i_1)}, \ldots, \widehat{\sigma(i_k)})$ на два упорядоченных набора чисел $(\widehat{\sigma(i_1)}, \ldots, \widehat{\sigma(i_m)})$ и $(\widehat{\sigma(i_{m+1})}, \ldots, \widehat{\sigma(i_k)})$, $1 \leqslant m \leqslant k-1$.

При $k = 1$ соотношения (2) записываются в виде $d(\partial_{(i)}) = 0$. Это говорит о том, что отображения $\partial_{(i)} : X_{n,\bullet} \to X_{n-1,\bullet}$, $0 \leqslant i \leqslant n$, являются отображениями дифференциальных модулей. При $k = 2$ соотношения (2) записываются в виде

$$d(\partial_{(i,j)}) = \partial_{(j-1)}\partial_{(i)} - \partial_{(i)}\partial_{(j)}, \quad i < j.$$

Это означает, что отображение $\partial_{(i,j)} : X_{n,\bullet} \to X_{n-2,\bullet+1}$ является гомотопией между отображениями дифференциальных модулей $\partial_{(j-1)}\partial_{(i)}, \partial_{(i)}\partial_{(j)} : X_{n,\bullet} \to X_{n-2,\bullet}$ и, следовательно, отображения $\partial_{(i)} : X_{n,\bullet} \to X_{n-1,\bullet}$ удовлетворяют с точностью до гомотопии симплициальным коммутационным соотношениям (1).

Большое количество содержательных примеров дифференциальных модулей с F_∞-симплициальными гранями доставляют дифференциальные A_∞-алгебры. В самом деле, пусть задана любая дифференциальная A_∞-алгебра $(A, d, \pi(n))$, где $\pi(n) : ((SA)^{\otimes n})_\bullet \to (SA)_{\bullet-1}$, $n \geqslant 2$, и SA – надстройка над A. Если положить $X = \{X_{n,m}\}$, где $X_{n,m} = ((SA)^{\otimes n})_{n+m}$, и определить F_∞-симплициальные грани $\widetilde{\partial} = \{\partial_{(i_1,\ldots,i_k)} : X_{n,\bullet} \to X_{n-k,\bullet+k-1}\}$ равенствами

$$\partial_{(i,i+1,\ldots,i+k-1)} = (-1)^\varepsilon 1^{\otimes(i-1)} \otimes \pi(k+1) \otimes 1^{\otimes(n-k-i)}, \quad 1 \leqslant i \leqslant n-k,$$

и $\partial_{(i_1,\ldots,i_k)} = 0$ в противном случае, то получим дифференциальный модуль с F_∞-симплициальными гранями $(X, d, \widetilde{\partial})$.

Для дифференциальных модулей с F_∞-симплициальными гранями определены их морфизмы и гомотопии между морфизмами. Доказана теорема о гомотопической инвариантности структуры дифференциального модуля с F_∞-симплициальными гранями, которая обобщает классическую теорему о гомотопической инвариантности структуры дифференциальной A_∞-алгебры.

Различные нормированные пространства, обладающие равными наборами локально минимальных сетей

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Локально минимальные сети на многообразиях и банаховых пространствах изучаются сравнительно давно, и есть несколько общеизвестных результатов. Например, в $\mathbb{R}^n$ с Евклидовой нормой любая локально минимальная сеть удовлетворяет следующим условиям:
1) все ребра вкладываются как отрезки
2) все подвижные вершины имеют степень два или три, и если степень вершины равна двум, то угол между инцидентными подвижной вершине ребрами равен 180^o, а если трем, то три инцидентных подвижной вершине ребра лежат в одной плоскости под углами 120^o друг к другу
3) граничные вершины имеют степень один, два или три, причем, если степень вершины равна двум, то два инцидентных ей ребра сходятся под углом $\geq 120^o$, а если степень вершины равна трем, то, как и в случае с подвижной вершиной, три инцидентных вершине ребра лежат в одной плоскости под углами 120^o друг к другу

Известна локальная структура локально минимальных сетей и для пространства $\mathbb{R}^n$ с Манхэттенской нормой: в нем подвижные вершины являются медианами трех соединяемых точек (то есть i-тая координата подвижной точки является медианой i-тых координат соединяемых точек). В данной работе изучается обратный вопрос, а именно вопрос восстановления нормы при известном локальном строении локально минимальных сетей. Оказывается, что пространство $\mathbb{R}^n, n \geq 3$ с Евклидовой нормой обладает уникальным семейством локально минимальных сетей (то есть не существует других норм на $\mathbb{R}^n$, исключая гомотетичные данной Евклидовой, которые обладали бы тем же семейством локально минимальных сетей, что и пространство с Евклидовой нормой). В то же время, в пространстве $\mathbb{R}^2$ с Евклидовой нормой это не так. Затем будет приведена серия примеров норм в пространстве $\mathbb{R}^2$, таких, что эти нормированные пространства обладают уникальными наборами локально минимальных сетей. Также в работе будет показано, что $\mathbb{R}^n, n \geq 2$ с Манхэттенской нормой обладает неуникальным семейством локально минимальных сетей (здесь будет использовано более узкое определение локальной миимальности сети, а именно, будут рассматриваться только те сети, ребра которых являются отрезками). Будет приведен пример пространства, не являющийся линейным преобразованием Манхэттенского, обладающего тем же набором локально минимальных сетей.

Parallehedrons, arising from convex hulls of orbits Weyl group of irreducible root systems

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n-мерный выпуклый многогранник в евклидовом пространстве E называется параллелоэдром, если в E найдется такое подмножество L, что 1) $E = \bigcup_{\nu \in L} (P+\nu)$ и 2) для любых $\nu_1, \nu_2 \in L$ пересечение $(P+\nu_1) \cap (P+\nu_2)$ — общая грань многогранников $(P+\nu_1)$ и $(P+\nu_2)$. Ряд глубоких структурных свойств параллелоэдров установил Г. Минковский (см. [1]):

Теорема 1. Если P — параллелоэдр в пространстве , то: а) P центрально-симметричен, б) гиперграни P центрально-симметричны, в) P имеет не более чем $2(2^n - 1)$ гиперграней, г) ортогональная проекция P, параллельно любой его грани коразмерности 2 на двумерную плоскость, является либо параллелограммом, либо центрально-симметричным шестиугольником. Позднее Б. А. Венков доказал, что указанные выше условия а), б) и г) характеризуют параллелоэдры (см. [1]). Весьма важные результаты о параллелоэдрах получены Б. Н. Делоне и его школой. В [2] было доказано, что многогранник P, являющийся выпуклой оболочкой $convO_a$ орбиты O_a вектора $a = (a_1, a_2, ..., a_n) \in E$ относительно стандартного действия на E группы подстановок S_n, является параллелоэдром тогда и только тогда, когда координаты a_j вектора a образуют возрастающую арифметическую прогрессию. Группа S_n является группой Вейля системы корней типа A_{n-1}. Естественно обобщить, рассмотренную в [2] задачу, на другие неприводимые системы корней кристаллографического типа. Точнее, верна

Теорема 2. Выпуклые оболочки орбит общего положения групп Вейля, неприводимых кристаллографических систем корней, отличных от системы корней типа A_n, не являются параллелоэдрами.

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Минимально-линейные вложения графов

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В докладе речь пойдет о таких вложениях графов в евклидово пространство, при которых наименьшее число точек образа принадлежит линейному подпространству, то есть, в общем случае, задача состоит в поиске таких вложений графов в n–мерное евклидово пространство, при которых максимальное число точек, принадлежащих одной k–мерной плоскости, минимально.

В первой части доклада будет рассмотрен случай пересечения образа графа прямой в трехмерном пространстве. Будет описано семейство минимальных графов, при каждом вложении которых в трехмерное пространство некоторая прямая содержит четыре точки образа.

Вторая часть доклада будет посвящена числу гиперпланарности графов — минимальному числу точек образа, принадлежащих гиперплоскости, при вложении в евклидово пространство заданной размерности. Будет описана верхняя оценка этого числа для деревьев, ассимтотически совпадающая с нижней.

В третьей части доклада пойдет речь о пересечении образа графа двумерной плоскостью при вложениях в четырехмерное пространство. Будет доказано, что любой планарный граф можно вложить в четырехмерное пространство так, что любая двумерная плоскость содержит не более четырех точек образа. Также будет приведено несколько примеров “экономичных” вложений графов.

Критерий аддитивности конечного метрического пространства и минимальные заполнения

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Минимальные заполнения конечных метрических пространств являются частным случаем обобщения проблемы Громова о минимальных заполнениях на стратифицированные многообразия. Рассматриваемая проблема имеет самостоятельный интерес и может быть представлена также как обобщение другой классической задачи, а именно, проблемы Штейнера о поиске кратчайшей сети, соединяющей заданные терминалы.

В теории минимальных заполнений псевдометрических пространств важную роль играют так называемые аддитивные пространства. Эти пространства также часто встречаются в приложениях, таких как биоинформатика, теория эволюции.

В докладе будет доказан следующий критерий аддитивности конечного метрического пространства:

Вес минимального заполнения псевдометрического пространства равен полупериметру этого пространства тогда и только тогда, когда пространство аддитивно.

Перечисление групп симметрий бифуркаций малой сложности в слоениях Лиувилля

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В теории гамильтоновых систем возникает задача изучения симметрий бифуркаций. В качестве бифуркаций далее рассматриваются особенности сложных функций Морса. Для их описания А.А. Ошемковым было введено понятие f-графа и предложен и релизован алгоритм кодировки f-графов, в результате работы которого был получен список f-графов малой сложности. Изучение симметрий особенностей функций Морса эквивалентно изучению симметрий f-графов. Автором был разработан алгоритм, распознающий алгоритмически симметрии заданного

f-графа. Для случая графов сложности 1,2,3 группы симметрий описаны в ([1]). Обозначим a_n - число f-графов фиксированной сложности, группа симметрий которых имеет порядок n.

В результате работы алгоритма, для атомов сложности 4,5,6,7 получаем:

Теорема 1 *Существует 62 неэквивалентных атома сложности 4, из них 20 имеют нетривиальные группы симметрий и следующий тип: 8 атомов типа* $(0, \mathbb{Z}_2)$; *5 атомов типа* $(1, \mathbb{Z}_2)$; *1 атом типа* $(0, \mathbb{Z}_4)$; *1 атом типа* $(0, \mathbb{Z}_4 \leftthreetimes \mathbb{Z}_2)$; *1 атом типа* $(1, \mathbb{Z}_4)$; *1 атом типа* $(1, \mathbb{Z}_2 \bigoplus \mathbb{Z}_2)$; *1 атом типа* $(1, \mathbb{Z}_4 \bigoplus \mathbb{Z}_2)$; *1 атом типа* $(1, \mathbb{Z}_8)$; *1 атом типа* $(2, \mathbb{Z}_2)$; *Существует 870 неэквивалентных f-графа сложности 5, из них 102 имеют нетривиальные группы симметрий:* $a_2 = 94$,;$a_5 = 4$ *;* $a_{10} = 4$*;Существует 9436 неэквивалентных f-графа сложности 6, из них 435 имеют нетривиальные группы симметрий:*$a_2 = 365$; $a_3 = 23$; $a_4 = 27$*;* $a_{12} = 15$*; Существует 122840 неэквивалентных f-графа сложности 7, из них 1617 имеют нетривиальные группы симметрий:* $a_2 = 1697$; $a_7 = 6$; $a_{14} = 4$*;*

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О симплициальной размерности

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Пусть K — конечный симплициальный комплекс. Для семейства $\Phi = \{F_1, \ldots, F_m\}$ замкнутых подмножеств нормального пространства X, для которого нерв $N(\Phi) \subset K$, определяется K-перегородка P семейства Φ следующим образом:

$$P = X \setminus V_1 \cup \cdots \cup V_m,$$

где V_j — окрестности F_j и $N(V_1, \ldots, V_m) \subset K$.

Имея K-перегородки можно определить размерности $K - \dim X$ и $K - \mathrm{Ind} X$. Если $K = \{0, 1\}$, то $K - \dim = \dim$, $K - \mathrm{Ind} = \mathrm{Ind}$. Размерности $K - \dim$ и $K - \mathrm{Ind}$ обладают многими свойствами классических размерностей dim и Imd, но имеется и своя специфика.