Asymptotic analysis of autoresonance in the system with small dissipation

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Outline

Statement of the problem

What about equations?

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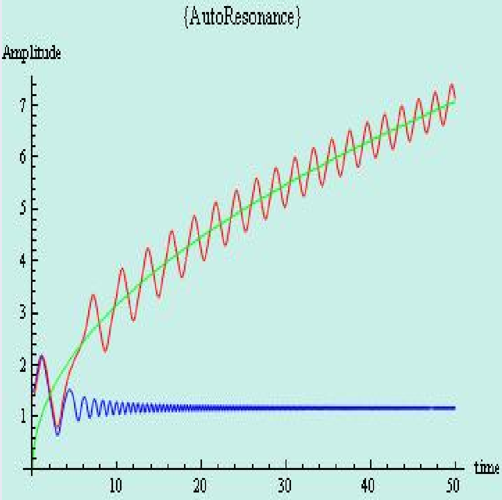
Object

The solution with increasing amplitude $\rho(t) \to \infty$ as $t \to \infty$, which is stable

Initial stage of autoresonance. Zero dissipation $\beta = 0$.

Constant driver amplitude: $f_1 = 0$. Autoresonance solution.



Model systems. First example.

Perturbed pendulum

$$\frac{d\rho}{dt} = \sin\Psi, \quad \frac{d\Psi}{dt} = \rho - \lambda t, \;\; \lambda = \mathrm{const} \neq 0.$$

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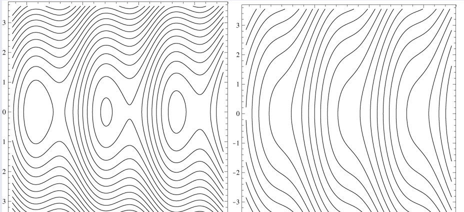
Phase portrait on the $(\Psi, \dot\Psi)$ plain under different λ



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Perturbed dissipationless pendulum

$$\frac{d\rho}{dt} = \sin\Psi, \quad \frac{d\Psi}{dt} = \rho - \lambda t.$$

Dissipation effect

$$\frac{d\rho}{dt} = \sin\Psi - \gamma\rho, \quad \frac{d\Psi}{dt} = \rho - \lambda t.$$

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Growing pumping & dissipation effect

$$\frac{d\rho}{dt} = (f_0 + f_1 t)\sin\Psi - \gamma\rho, \quad \frac{d\Psi}{dt} = \rho - \lambda t, \quad (f_0, f_1 = \mathrm{const}).$$

Model systems. Second example.

Perturbed pendulum

$$\frac{d\rho}{dt} = \sin\Psi, \quad \frac{d\Psi}{dt} = \rho - \lambda t.$$

Main autoresonance equations

$$\frac{d\rho}{dt} = \sin\Psi, \quad \rho\left[\frac{d\Psi}{dt} - \rho^2 + \lambda t\right] = b\cos\Psi, \quad (b = \mathrm{const}).$$

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What is the autoresonance

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$$\frac{d^2x}{dt^2} + \sin x = 0$$

Phase portrait of the unperturbed pendulum

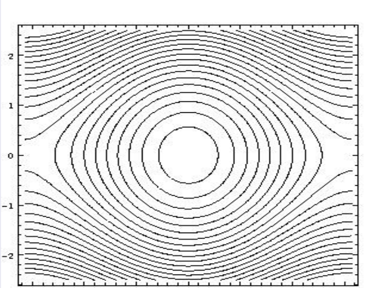

Figure: Pendulum trajectories on the $(x, \dot{x})$ plane

Problem of the perturbed pendulum dynamic

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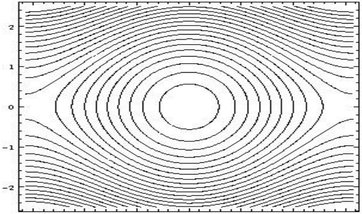
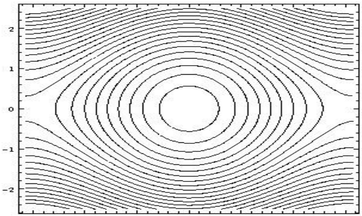


Problem of the perturbed pendulum dynamic



Is it possible to reach large amplitude oscillation $x \approx 1$ starting near equilibrium $x = \dot{x} = 0$ and using a weak pumping $0 < \varepsilon \ll 1$ under small dissipation $0 < \Gamma \ll 1$?

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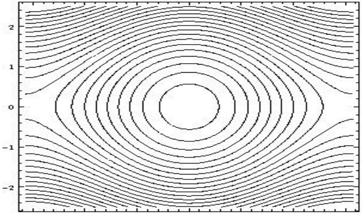

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Naive resonance perturbation of the pendulum

$$\frac{d^2x}{dt^2} + \sin x = -\Gamma \frac{dx}{dt} + \varepsilon \cos t.$$

Problem of the perturbed pendulum dynamic

Specific of the nonlinear oscillator

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Advanced perturbation of the pendulum

$$\frac{d^2x}{dt^2} + \sin x = -\Gamma \frac{dx}{dt} + \varepsilon(1 + \delta t)\cos(t - \alpha t^2).$$

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Perturbed equation is not integrable.

Solution is analyzed by either numerical simulation or asymptotic methods.

Pendulum under oscillating pumping. Simulation.

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Equation

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Small parameters

$$\varepsilon = 10^{-4},\ \Gamma = 0 \div 10^{-4},\ \delta = 0 \div 10^{-4},\ \alpha = 0 \div 3\cdot 10^{-6}.$$

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Input: Small initial data

$$[\dot{x}^2 + x^2]_{t=0} \le \varepsilon^{2/3}.$$

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Output: Energy

$$E = \frac{1}{2}\dot{x}^2(t) + 1 - \cos x(t), \quad 0 < t \le \mathcal{O}(\varepsilon^{-1}).$$

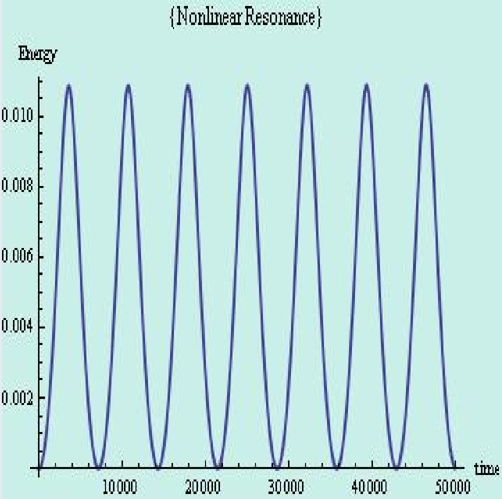
Nonlinear resonance – driver frequency is constant: $\alpha = 0$

Constant driver amplitude.

$$\frac{d^2x}{dt^2} + \sin x = -\Gamma \frac{dx}{dt} + \varepsilon \cos t.$$

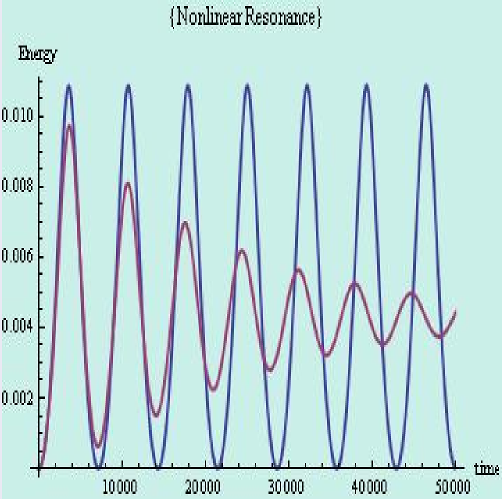
Nonlinear resonance – driver frequency is constant: $\alpha = 0$

Constant driver amplitude. Zero dissipation $\Gamma = 0$



Nonlinear resonance – driver frequency is constant: $\alpha = 0$

Constant driver amplitude. Effect of dissipation $\Gamma = \varepsilon$.



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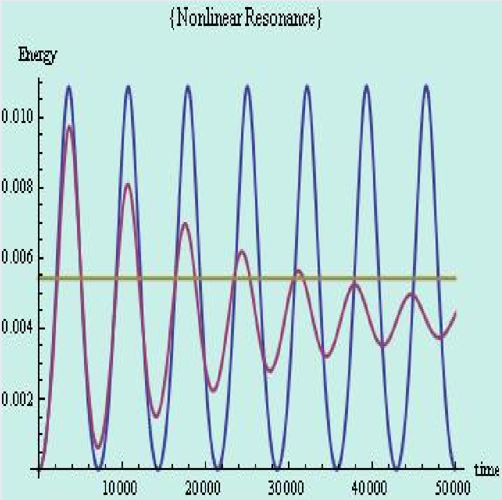
Constant driver amplitude. Adiabatic approximation.



Nonlinear resonance – driver frequency is constant: $\alpha = 0$

Increasing driver amplitude $\varepsilon + \varepsilon^2 t$.

$$\frac{d^2 x}{dt^2} + \sin x = -\Gamma \frac{dx}{dt} + \varepsilon(1 + \varepsilon t)\cos t.$$

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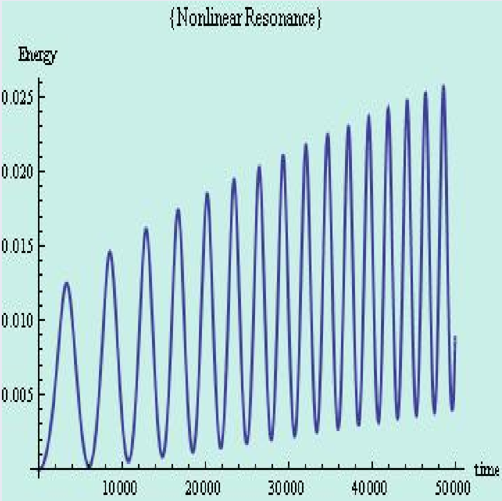
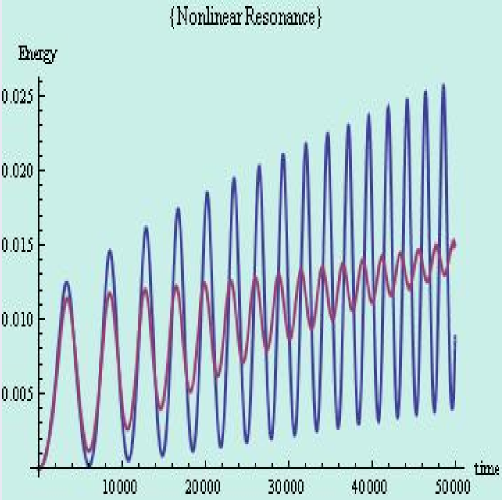
Increasing driver amplitude $\varepsilon + \varepsilon^2 t$. Zero dissipation $\Gamma = 0$



Nonlinear resonance – driver frequency is constant: $\alpha = 0$

Increasing driver $\varepsilon + \varepsilon^2 t$. Effect of dissipation $\Gamma = \varepsilon$.



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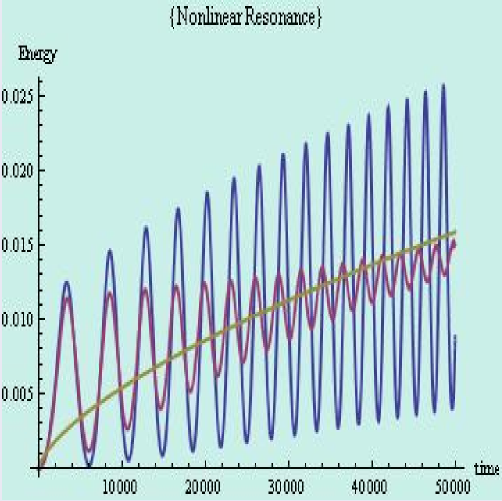
Increasing driver. Adiabatic approximation.



How to get large energy under weak pumping?

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Conclusion from simulation.

It is impossible to get large energy by using a constant driver frequency.

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- To vary the driver frequency in order to keep the resonance with the free frequency while the energy begins to grow.

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Simulation the autoresonance. Zero dissipation.

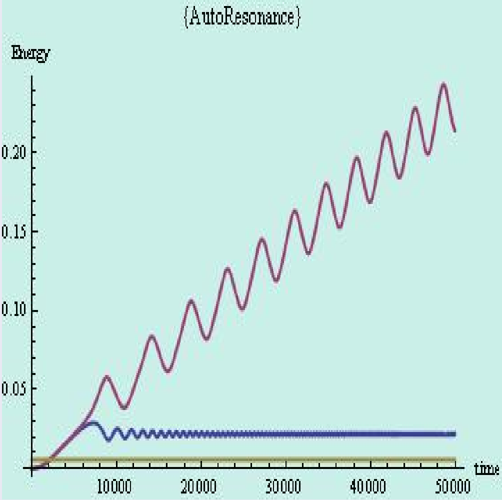
Pendulum under oscillating pumping

$$\frac{d^2x}{dt^2} + \sin x = \varepsilon \cos(t - \alpha t^2).$$

Variation of the pumping frequency as $\alpha > 0$.

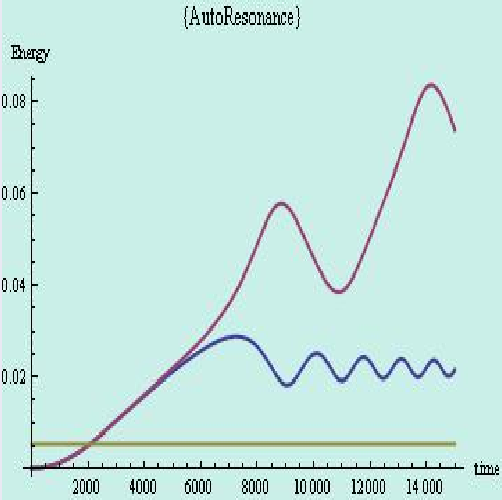
Dissipationless systems. Decreasing driver frequency: $1 - 6 \cdot 10^{-6} t$

Constant driver amplitude $\varepsilon = 10^{-4}$. Two type of solutions.



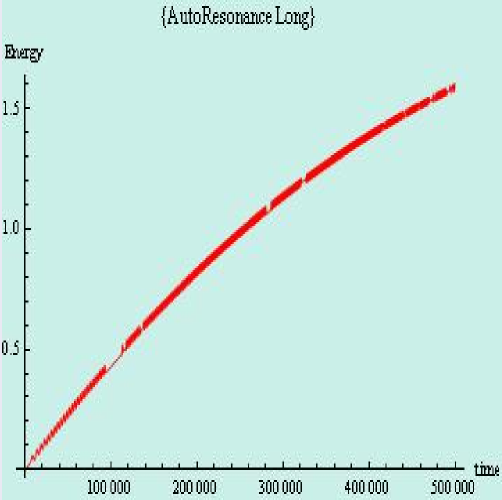
Dissipationless systems. Decreasing driver frequency: $1 - 6 \cdot 10^{-6} t$

Constant driver amplitude $\varepsilon = 10^{-4}$. Initial stage.



Dissipationless systems. Decreasing driver frequency: $1 - 6 \cdot 10^{-6} t$

Constant driver amplitude $\varepsilon = 10^{-4}$. Long times.



Dissipation systems.

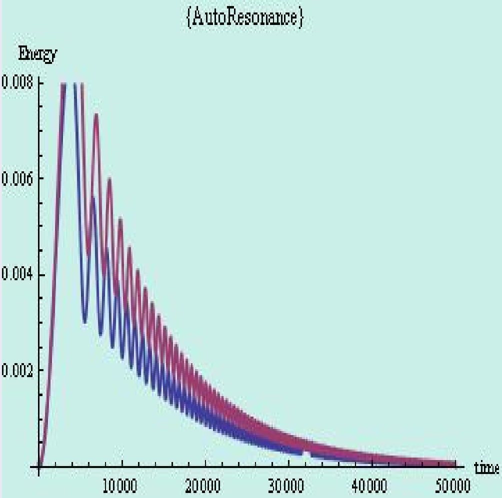
Dissipation systems.

$$\frac{d^2x}{dt^2} + \sin x = -\Gamma \frac{dx}{dt} + \varepsilon \cos(t - \alpha t^2), \quad \Gamma \approx \varepsilon > 0.$$

Is autoresonance possible in dissipation system?

Dissipation systems. Decreasing driver frequency: $1 - 6 \cdot 10^{-6} t$

Constant driver amplitude. Effect of dissipation $\Gamma = \varepsilon$.



How to get large energy in dissipation system under $\Gamma > 0$

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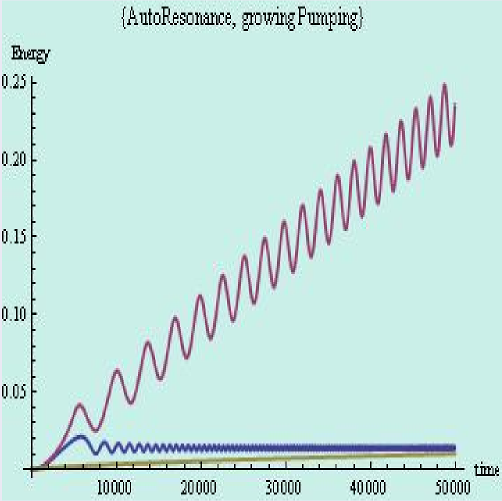
References

L.A.Kalyakin, M.A. Shamsutdinov. Autoresonant asymptotics in the oscillating system with weak dissipation . TMPh. (2009), 160, 1, p.960-967.

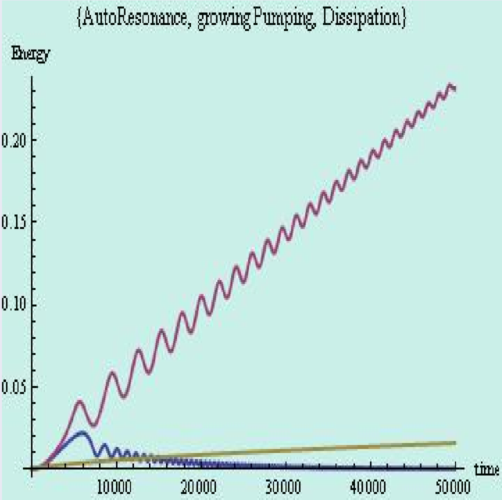
Decreasing driver frequency: $1 - 6 \cdot 10^{-6} t$

Increasing driver amplitude: $\varepsilon + 10^{-5} t$. Zero dissipation.



Decreasing driver frequency: $1 - 6 \cdot 10^{-6} t$

Increasing driver amplitude: $\varepsilon + 10^{-4} t$. Dissipation effect.



The problem of separation

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Conclusion from simulations

The autoresonance phenomenon depends on the initial data.

The problem of separation

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Main problem in autoresonance theory

The problem of separation

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Main problem in autoresonance theory

How to separate the different types of solutions?

The problem of separation

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How to separate the different types of solutions?
Suggest: Analysis on the initial stage.

Analytical results for pendulum

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An example of nonlinear oscillator under perturbation

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Anzatz. Asymptotic approximation on the initial stage

$$x(t;\varepsilon) = \varepsilon^{1/3}\frac{1}{2}\rho(\tau)\cos(t-\alpha t^2-\Psi(\tau)) + \mathcal{O}(\varepsilon^{2/3});\ \tau=\varepsilon^{2/3}t.$$

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Result of averaging

$$\frac{d\rho}{d\tau} = f(\tau)\sin\Psi - \gamma\rho, \quad \rho\left[\frac{d\Psi}{d\tau} + \lambda\tau - \rho^2\right] = f(\tau)\cos\Psi.$$

Asymptotic approximation on the initial stage

Original equation

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- $\lambda = 2\alpha/\varepsilon^{4/3}$
- $\gamma = \Gamma/2\varepsilon.$

Asymptotic approximation on the initial stage

Original equation

$$\frac{d^2x}{dt^2} + x - x^3/6 = -\Gamma\frac{dx}{dt} + \varepsilon(1 + \delta t)\cos(t - \alpha t^2).$$

Result of averaging

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- $f(\tau) = 1 + (\delta/\varepsilon^{2/3})\tau$
- $\lambda = 2\alpha/\varepsilon^{4/3}$
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Asymptotic approximation on the initial stage

Anzatz on the initial stage

$$x(t;\varepsilon) = \varepsilon^{1/3}\frac{1}{2}\rho(\tau)\cos(t - \alpha t^2 - \Psi(\tau)) + \mathcal{O}(\varepsilon^{2/3});\ \tau = \varepsilon^{2/3}t.$$

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- Are they stable?

Analytical results. Dissipationless system.

$$\frac{d\rho}{d\tau} = f\sin\Psi, \quad \rho\Big[\frac{d\Psi}{d\tau} + \lambda\tau - \rho^2\Big] = f\cos\Psi, \ \lambda = \mathrm{const} > 0.$$

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- Autoresonance solutions.
  Let be $\gamma = 0, \ f = \text{const}$. If $\lambda > 0$ then there exist two-parametric solution, which has the increasing amplitude $\rho(\tau) = \sqrt{\lambda\tau} + \mathcal{O}(\tau^{-3/8}), \ \tau \to \infty$.

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L.A.Kalyakin, Russian Math. Surveys. 63, 5 (2008). 3-72.

Separation of the autoresonance solutions

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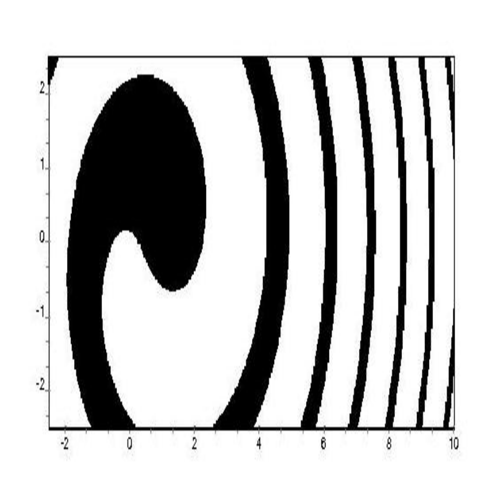
- There are some numerical results.

Separation of the autoresonance solutions

The phase plain at the initial moment (R.Garifullin, 2003)



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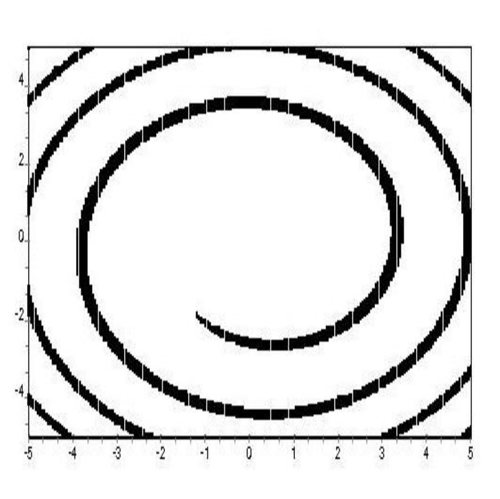

Dissipation systems.

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$$\frac{d^2x}{dt^2} + \sin x = -\Gamma \frac{dx}{dt} + \varepsilon \cos(t - \alpha t^2), \; \Gamma \approx \varepsilon > 0.$$

Is autoresonance possible in dissipation system?

Dissipation system under a constant pumping amplitude

Main resonance equations

$$\frac{d\rho}{d\tau} = f(\tau)\sin\Psi - \gamma\rho, \quad \rho\left[\frac{d\Psi}{d\tau} + \lambda\tau - \rho^2\right] = g(\tau)\cos\Psi, \ \gamma, \lambda > 0.$$

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Theorem.

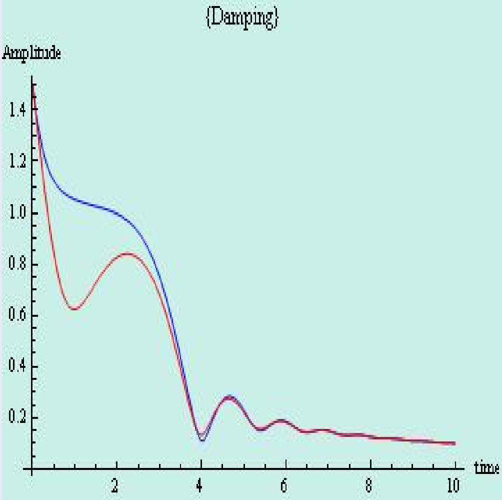
Let be $f = \mathrm{const}$. If $\gamma > 0$ then the amplitude of each solution is bounded.

Corollary.

Autoresonance phenomenon can not be in any dissipation system under constant driver amplitude.

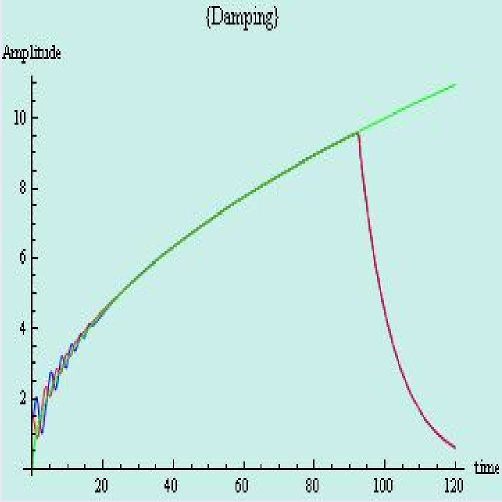
Constant pumping amplitude. Damping under dissipation

Strong dissipation $\gamma = 1$



Constant pumping amplitude. Damping under dissipation

Weak dissipation $0 < \gamma \ll 1$



Resonance under dissipation $\Gamma > 0$. Idea.

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References: L.A.Kalyakin, M.A. Shamsutdinov, Theor. Math. Phys. (2009), 160, 1.

Dissipation system under the varying pumping amplitude

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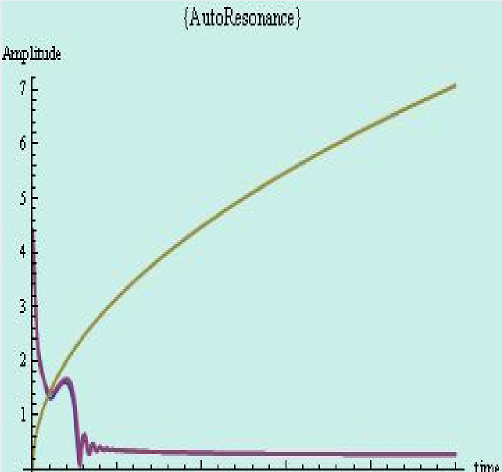
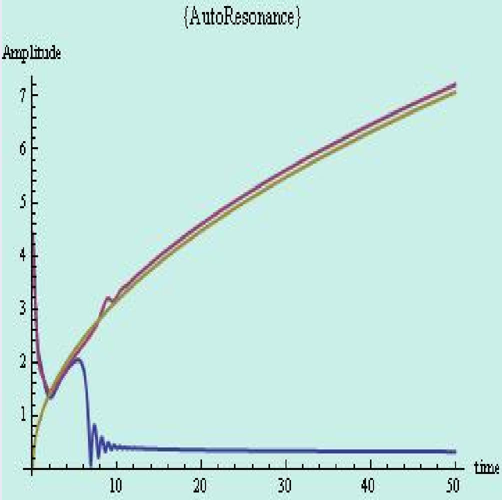
Autoresonance and dissipation

Increase of the driver amplitude: $f_1 = 0.25$. Nonautoresonance.



Autoresonance and dissipation

Increase of the driver amplitude: $f_1 = 0.3$. Autoresonance.



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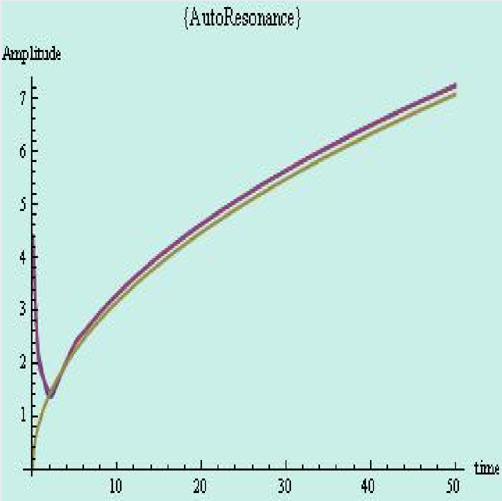

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- Increase of the small driver amplitude is necessary for existence of this phenomenon.
- Increase of the output amplitude is determined by variation of the driver frequency.
- Ensemble of the autoresonance solutions depends on the rate of growth of the driver amplitude.

Dificulty of the asymptotics

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Back to the main resonance equations

$$\frac{d\rho}{dt} = \sin\Psi, \quad \rho\Big[\frac{d\Psi}{dt} - \rho^2 + \lambda t\Big] = \cos\Psi.$$

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Two different solutions

$\sin\Psi_0 = 0 \Rightarrow$ either $\Psi_0 = 0$ or $\Psi_0 = \pi$.
There are no any integration constants in the expansions.

Asymptotics is a beautiful and very hard task

Two parametric WKB-type asymptotics

$$\rho(t;c,s_0) = \sqrt{\lambda t} + t^{-1/4}\sum_{n=1}^{\infty} t^{-n/8}\rho_n(S;c),$$

$$\Psi(t;c,s_0) = \pi + \sum_{n=1}^{\infty} t^{-n/8}\Psi_n(S;c), \quad t\to\infty.$$

$$S = (4/5)\sqrt{2}\lambda^{1/4}t^{5/4} + s_{0,1}(c)\ln t + s_0$$

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Construction of the WKB-asymptotics is very hard. It is the art in some sense (V. Babich)

Difficulty of the asymptotics

Equations of degenerate resonance

$$\frac{d\rho}{dt} = \sin\Psi, \quad \frac{d\Psi}{dt} = \rho^2 - \lambda t.$$

There are no any WKB-asymptotics at all.

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$$\frac{d\rho}{dt} = (f_0 + f_1 t)\sin\Psi - \gamma\rho, \quad \frac{d\Psi}{dt} = \rho^2 - \lambda t.$$

WKB-asymptotics ???

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$$\frac{d\rho}{dt} = \rho \sin \Psi, \quad \frac{d\Psi}{dt} - \rho + \lambda t = b \cos \Psi.$$

First correction WKB-asymptotics is known.

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WKB-asymptotics ???

Stability of the autoresonance

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There is only interest in the stable solutions for applications. Those solutions correspond to physically realizable states.

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Skill of Lyapunov functions instead of skill of asymptotics

Idea is to prove stability of a single power solution instead the general WKB-asymptotics.

This approach is more preferable for physicists, because it allows to take into account the random perturbation.

Example: Bloch equations

$$\frac{d\rho}{dt} = -t\,\sin\Psi - \beta_2\rho, \quad \frac{dz}{dt} = t\,\rho\,\sin\Psi - \beta_1 z$$

$$\rho\Big[\frac{d\Psi}{dt} + \lambda t - z\Big] = -t\,\cos\Psi; \quad (\lambda, \beta_1, \beta_2 = \mathrm{const} > 0).$$

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Bad idea is to construct three parametric WKB-asymptotics with growing amplitude.

Alternative approach is to prove the stability of a single solution with growing amplitude.

Averaged Bloch equations

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Power asymptotics

$$\rho(t) = r_1\sqrt{t} + r_0 + \sum_{n=1}^{\infty} \rho_{-n} t^{-n/2},$$

$$z(t) = \lambda t + z_1\sqrt{t} + z_0 + \sum_{n=1}^{\infty} z_{-n} t^{-n/2},$$

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Leading order term

$\sin\Psi_0 = 0 \ \Rightarrow \ \Psi_0 = \pi$, or $\Psi_0 = 0 \ \Rightarrow$ There are two asymptotic solutions with growing amplitude.

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What solution is stable?

Main result

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Denote the solution $R_0(t),\ Z_0(t).\ \Psi_0(t)$ which is determined by the leading order term $\Psi_0 = 0$.

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$$z(t) = \lambda t + z_1\sqrt{t} + z_0 + \sum_{n=1}^{\infty} z_{-n} t^{-n/2},$$

$$\Psi(t) = \Psi_0 + \sum_{n=1}^{\infty} \Psi_{-n} t^{-n/2}.$$

Denote the solution $R_0(t),\ Z_0(t).\ \Psi_0(t)$ which is determined by the leading order term $\Psi_0 = 0$.

Theorem

If $\beta_1, \beta_2 > 0$ then the solution $R_0(t),\ Z_0(t).\ \Psi_0(t)$ is asymptotically stable as $t \to \infty$.

Main result

Construction of a Lyapunov's function for the remainder problem is the idea of the proof.

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References

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Future: Stability of the power solutions for different autoresonance models.

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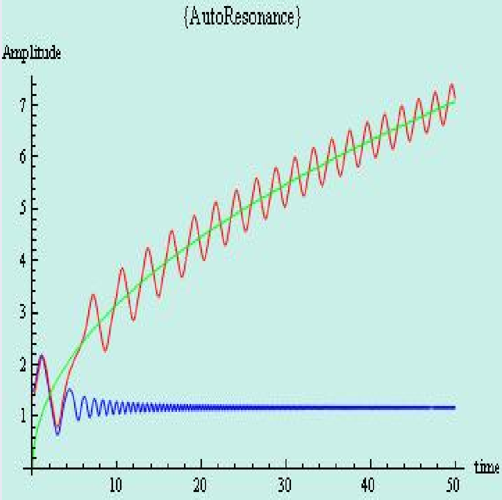
Future activity

Stability of autoresonance models under random perturbation

Initial stage of autoresonance. Zero dissipation $\beta = 0$.

Constant driver amplitude: $f_1 = 0$. Autoresonance solution.



Thanks

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